Interval Computations
No 4(6), 1992

GENERAL PROBLEM OF THE ASYMPTOTIC STEADY-OUTPUT TRACKING FOR PLANT WITH INTERVAL PARAMETERS

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The solution of the output asymptotic steady-tracking problem with the unmeasurable input and output disturbances rejection for interval parameters plant is presented. The solving PI-regulator with the state estimator is proposed. The general solving conditions for this problem is received. This conditions are reduced to the analysis of the interval system controllability and observability and to the restrictions for special block interval matrices.

ОБЩАЯ ЗАДАЧА АСИМПТОТИЧЕСКОГО СЛЕЖЕНИЯ ЗА КУСОЧНО-ПОСТОЯННЫМ СИГНАЛОМ ДЛЯ ОБЪЕКТА С ИНТЕРВАЛЬНЫМИ ПАРАМЕТРАМИ

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Представлено решение проблемы асимптотического слежения для объекта с интервальными параметрами при наличии неизмеряемых возмущений на входе и выходе. Для решения проблемы используется ПИ-регулятор по оценке состояния. Получены общие условия разрешимости проблемы. Эти условия сводятся к анализу управляемости и наблюдаемости интервальной системы и ограничениям на специально сформированные блочные интервальные матрицы.

1. Introduction

The method of the multivariable PI-regulator designing to solve the asymptotic steady-tracking problem for linear time-invariant dynamical system having the constant parameters with a unmeasurable regulated output and unmeasurable piece-step disturbances in the input and output had been considered in [1]. For this case the difference between the regulated output estimation and the reference signal is used in the PI-regulator structure.

Now we consider the similar problem for a linear time-invariant dynamical system with parameters being uncertained within some bounds. The solving conditions for this problem have been received. This conditions generalize the corresponding ones from the paper [1]. The present paper is a continuation of the asymptotic regulation researches for interval systems [2,3].

2. Problem statement

Consider the following linear time-invariant interval system

$$\dot{x} = [A]x + [B]u + [E]w \tag{1}$$

with the measurable output

$$y = [H]x + [F]w \tag{2}$$

and a unmeasurable regulated output

$$z = [D]x \tag{3}$$

where $x \in R^n$ is a state vector, $u \in R^r$ is an input vector, $y \in R^l$, $z \in R^m$. The vector $w \in R^d$ is an unmeasurable disturbance, described by the linear dynamical system

$$\dot{w}(t) = 0, \quad w(t_i) = w_{0i} = \text{const}, \quad t \neq t_i, \; i = 0, 1, \ldots \tag{4}$$

with unknown w_{0i}. So, w is an unmeasurable piece-step function changing during the system working in any time. The matrices $[A]$, $[B]$, $[H]$, $[E]$, $[F]$, $[D]$ are interval matrices with elements having the following bounds:

$$\begin{gathered} a_{ij} = [a_{0ij}, a_{bij}], b_{ik} = [b_{0ik}, b_{bik}], h_{pj} = [h_{0pj}, h_{bpj}], e_{is} = [e_{0is}, e_{bis}], \\ f_{ps} = [f_{0ps}, f_{bps}], d_{tj} = [d_{0tj}, d_{btj}], i, j = 1, \ldots, n, \; k = 1, \ldots, r, \\ p = 1, \ldots, l, \; s = 1, \ldots, d, \; t = 1, \ldots, m. \end{gathered} \tag{5}$$

Let $z_0(t) = z_0,\ z_0 \in R^m$ is the desired reference signal which is described by the linear dynamical system

$$\dot{z}_0(t) = 0, z_0(t_i^*) = \text{const}, t \neq t_i^*, i = 0, 1, \ldots \tag{6}$$

with the known $z_0(t_i^*)$. So, z_0 is the measurable piece-step function changing during the system working in any time.

For the plant (1)–(5) it is required to find such a constant feedback control that in the closed-loop system asymptotic steady-output tracking

$$z \rightarrow z_0, \qquad t \rightarrow \infty \tag{7}$$

is ensured for all disturbance w and all parameters changing within the bounds (5).

3. Main result

We will use a feedback controller having the error $z_0 - \hat{z}$ as input. It has the form

$$\dot{q} = z_0 - \hat{z} = z_0 - [D]\hat{x}, \tag{8}$$

$$u = K_1\hat{x} + K_2 q, \tag{9}$$

where $q \in R^m, \hat{x} \in R^n$ is an estimation of the vector $x, \hat{z} = [D]\hat{x}, K_1, K_2$ are constant feedback gain matrices which have the sizes $r \times n, r \times m$.

In this problem we should estimate both the state vector x and the disturbance vector w. For the estimating of the augmented vector $[x^T, w^T]$ we can use the full order observer for the following model

$$\begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} [A] & [E] \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} [B] \\ 0 \end{bmatrix} u, \tag{10}$$

$$y = ([H], [F]) \begin{bmatrix} x \\ w \end{bmatrix}. \tag{11}$$

For (10),(11) the general equation for estimation vector $(\hat{x}^T, \hat{w}^T)$ may be written as

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{bmatrix} = \begin{bmatrix} [A] & [E] \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} + \begin{bmatrix} [B] \\ 0 \end{bmatrix} u - L([H], [F])\epsilon, \tag{12}$$

where L is an $(n+d)\times l$ matrix, $\epsilon^T \doteq (x^T, w^T)-(\hat{x}^T, \hat{w}^T)$ is an estimating error. It is easy to show that the vector ϵ satisfies to the set of the differential equations

$$\dot{\epsilon} = \left\{ \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - L(H, F) \right\} \epsilon, \tag{13}$$

where

$$A \in [A], \quad E \in [E], \quad H \in [H], \quad F \in [F]. \tag{14}$$

If the system (13) is asymptotic stable for every matrix (14) then $\epsilon \to 0$ as $t \to \infty$, i.e. $\hat{x} \to x, \hat{w} \to w$ as $t \to \infty$. So, we should find a constant matrix L ensured asymptotic stability of (13) for all matrix (14). Construct the matrix L so that the all characteristic polynomial [4] coefficients of the interval dynamic matrix of the system (13) would be located inside the appropriate interval $[p_i], i = 1, 2, \dots, n+d$ of the assigned asymptotic stable interval polynomial

$$[P(s)] = s^{n+d} + [p_{n+d-1}]s^{n+d-1} + \ldots + [p_1]s + [p_0]. \tag{15}$$

For obtaining of the observer matrix L we can use the method proposed in [5].

Then we should find the controller (8),(9) which we rewrite as

$$u = [K_1, K_2] \begin{bmatrix} \hat{x} \\ \hat{q} \end{bmatrix} = [K_1, K_2] \begin{bmatrix} x \\ q \end{bmatrix} - [K_1, 0]\epsilon, \tag{16}$$

$$\dot{q} = z_0 - \hat{z} = z_0 - [D]x + ([D], 0)\epsilon. \tag{17}$$

Using (1),(16),(17) we can write the augmented closed-loop

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} &= \begin{bmatrix} [A]+[B]K_1 & [B]K_2 \\ -[D] & 0 \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} + \begin{bmatrix} [E] \\ 0 \end{bmatrix} w \\ &+ \begin{bmatrix} [B]K_1 & 0 \\ [D] & 0 \end{bmatrix} \epsilon + \begin{bmatrix} 0 \\ -I_m \end{bmatrix} z_0. \end{aligned} \tag{18}$$

The constant matrix K_1, K_2 in (16) should be found that the all characteristic polynomial coefficients of the interval matrix

$$\begin{bmatrix} [A]+[B]K_1 & [B]K_2 \\ -[D] & 0 \end{bmatrix} \tag{19}$$

would be located inside the appropriate interval $[\bar{p}_i]$, $i = 1, 2, \ldots, n+m$ of the assigned asymptotic stable interval polynomial

$$[\bar{P}(s)] = s^{n+m} + [\bar{p}_{n+m-1}]s^{n+m-1} + \ldots + [\bar{p}_1]s + [\bar{p}_0]. \qquad (20)$$

For this purpose the same algorithm [5] can be employed.

It can be shown that for the united control system (18) and the error estimation system (13) with the state $[x^T, q^T, \epsilon^T]$ the following condition is satisfied $\dot{q} \to 0$ as $t \to \infty$ hence $z \to z_0$ as $t \to \infty$.

The analysis gives the following solving conditions of the problem:

1) pair $([A], [B])$ is controllable, pair $([H], [A])$ is observable,

2) $r \geq m$, $l \geq d$,

3) from the columns of the interval rectangular matrices

$$\begin{bmatrix} [A] & [B] \\ -[D] & 0 \end{bmatrix}, \quad \begin{bmatrix} [A]^T & [H]^T \\ [E]^T & [F]^T \end{bmatrix} \qquad (21)$$

the following square matrices $[Q_1], [Q_2]$ of the order $n+m$, $n+d$ can be formed for which

$$0 \notin \det[Q_1], \qquad 0 \notin \det[Q_2]. \qquad (22)$$

Consider the problem of the asymptotic steady-output tracking and disturbances rejection for the following system

$$\dot{x} = Ax + Bu + \overline{E}h(t), \qquad (23)$$

$$y = Hx, \qquad (24)$$

$$z = Dx \qquad (25)$$

with the constant parameters and an unmeasurable disturbance $h(t)$. The elements of $h(t)$ may change within bounds

$$m_{0i} \leq h_i(t) \leq m_{bi}, \; i = 1, \ldots, d. \qquad (26)$$

where m_{0i}, m_{bi} are known constants. The functions $h_i = h_i(t)$ satisfy the differential equations

$$\dot{h}_i = 0, \; m_{0i} \leq h_i(t_0) \leq m_{bi}.$$

It can be shown that the system (23)–(25) is described equivalently by the system (1)–(3) with the constant matrices $[A] = A$, $[B] = B$, $[H] = H$, $[F] = 0$, $[D] = D$ and $[E] = \overline{E}[M]$ where $\overline{E}$ is a constant matrix and $[M]=\text{diag}\{[m_1], [m_2], \ldots, [m_d]\}$ is a diagonal matrix with the interval diagonal elements $[m_i] = [m_{i0}, m_{ib}]$. For this system the condition (21) may be formulated as: $0 \notin [m_1][m_2]\ldots[m_d]$.

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