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# ALGORITHMIC AIMS OF RELIABILITY PROVISION FOR LARGE—SCALE DYNAMIC SYSTEMS WITH INTERVAL PARAMETERS

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In the present paper the problem of the fault-tolerant control algorithm design is considered. The dynamic behavior of a plant is described by the transfer matrices with interval parameters. The offered approach is based on the system of characteristic polynomials analysis and using of Kharitonov's theorems.

# АЛГОРИТМИЧЕСКИЕ ЦЕЛИ ОБЕСПЕЧЕНИЯ НАДЕЖНОСТИ ДЛЯ ШИРОКОМАСШТАБНЫХ ДИНАМИЧЕСКИХ СИСТЕМ С ИНТЕРВАЛЬНЫМИ ПАРАМЕТРАМИ

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В работе рассмотрена задача построения алгоритма управления, устойчивого к отказам. Динамическое поведение производства описано матрицами переноса с интервальными параметрами. Предлагаемый подход базируется на анализе системы характеристических полиномов и использовании теоремы Харитонова.

## 1. Introduction

In modern control systems development the first priority is given to a provision of two important system performances: reliability and robustness. The solution of security problem is greatly influenced by the response of the control algorithm to any fault.

As is well-known, any arbitrary algorithm includes a sequence of instructions for certain actions (operations) that give a possibility to realize a bridge from initial data set to a result to be found. Hence algorithmic failure can be considered as a random event making the achievement of the desirable result impossible. All failures of that type are caused by the following families of reasons:

(i) initial data perturbations and (or)

(ii) erroneous fulfilment of algorithmic instructions.

In the case of control algorithm, whose purpose is to form some special inputs of the dynamic plant, each of the above reasons may be connected with different events, such as: operation conditions unpredicted change, uncertainty in plant parameters, sudden or stage-by-stage hardware failures, computer malfunctions, software errors, etc. Thus fault-tolerant control algorithm must provide system accident-free action when one or more reasons that cause algorithmic failures take place.

The main ideas of the present paper are specially developed for the important class of modern control systems — the class of largescale systems with multi-input, multi-output plants [1]. Many technological processes, aircrafts and their engines are glowing examples of plants of this type. The characteristic feature of the systems to be considered is the distribution of the control problems between subsystems according to their functional abilities. Separate interconnected subsystems explore different physical phenomena and may be autonomously developed by the different designers. Under such conditions it is necessary to determine the desirable behavior of all the system parts for the cases of possible subsystems failures. The mentioned problem must be solved when a majority of system states has been taken into account.

## 2. The problem formulation

Let a large-scale dynamic system (LSDS) be composed of subsystems which belong to $M$ control levels. Each subsystem $G_i$, $i \in \Omega^{(k)} = \{\overline{1..N}\}$ from the $k$-th level is described by the equation

$$y_i^{(k)}(s) = W_i^{(k)}(s) \cdot u_i^{(k)}(s), \tag{1}$$

where $y_i^{(k)}(s)$, $u_i^{(k)}(s)$ are vectors of Laplace transformed outputs and inputs of $G_i$, $W_i^{(k)}(s)$ is the transfer matrix of $G_i$.

For the lower level (if $k = 1$)

$$\Omega^{(1)} = \Omega_R \cup \Omega_H, \quad \Omega_R \cap \Omega_H = \emptyset.$$

Here $\Omega_R$ is a set of subsystems numbers which belong to the controller, $\Omega_H$ is an analogue set with respect to the plant.

The interconnections of subsystems are described by the expressions:

$$\forall \left( r \in \Omega^{(k)} \wedge i \in \Omega_r^{(k-1)} \right) :$$
$$u_i^{(k-1)}(s) = \sum_{j \in \Omega_r^{(k-1)}} F_{ij}^{(k,r)} \cdot y_j^{(k-1)}(s) + R_i^{(k,r)} \cdot u_r^{(k)}(s), \tag{2}$$
$$y_r^{(k)}(s) = \sum_{i \in \Omega^{(k)}} L_i^{(k,r)} \cdot y_i^{(k-1)}(s),$$

where $\Omega_r^{(k-1)} \subset \Omega^{(k-1)}$ and $\bigcap\limits_r \Omega_r^{(k-1)} = \emptyset$, $F_{ij}^{(k,r)}$, $R_i^{(k,r)}$, $L_i^{(k,r)}$ — matrices, whose entries, belong to the discrete set $\Delta = \{-1, 0, +1\}$.

The model (1), (2) usually contains uncertain components, for example, uncertainties due to parameters, constant or variable, which are unknown or imperfectly known. In a general situation, it is known only, that some coefficients of a given part transfer functions belong to closed intervals with fixed bounds. In this case we can say that model (1), (2) describes interval system [2], [3].

Let $H_0$ be a state of a faultless system. The algorithmic failures of subsystems constrain components of their models, such as $W_i^{(k)}(s)$, to change. So it is possible to note a number of states of the system $H_1, \ldots, H_p$ being concerned with failures of different types. It is required to determine the structure and the parameters of the controller subsystems for each $k-$ th level with respect to the state $H_0$ of the system, and, in addition, to find all the entries of matrices in the equations (2) in order to provide the desirable degree of stability and dumping ratio for the states $H_1, H_2, \ldots, H_p$. The formulation of this problem is concerned with the provision of the interval systems admissible pole placement properties.

## 3. The design method

The design method offered is based on the replacements of a sequence of variables and the use of approach [4]. It is necessary to note that the results [4] allow to reduce the considered problem to the problem of the stability provision for the fixed number of characteristic polynomials with the real coefficients.

The entire procedure consists of two independent stages. At the first stage, the structure of all subsystems and their interconnections are determined. The solution is found as the result of algorithmic complexity cost function optimization [5].

At the second (parametric) stage, the entire system interval characteristic polynomials

$$[D(s, H_i)] = \left[d_0^{H_i}\right] s^n + \left[d_1^{H_i}\right] s^{n-1} + \ldots + \left[d_n^{H_i}\right] \qquad (3)$$

for all LSDS states $H_i$, $i \in \{\overline{0, p}\}$ are formed.

Here $[d_l^{H_i}]$, $l \in \{\overline{0, n}\}$ are interval coefficients connected with the parameters $\gamma_1, \gamma_2, \ldots, \gamma_\lambda$ of the control subsystems.

Then additional interval polynomials $[D(s^*, H_i)], [D(s^{**}, H_i)]$ are obtained form (3) by the replacements

$$s^* = s - \eta; \quad s^{**} = s \cdot \exp(-j\varphi), \qquad (4)$$

where $\eta$ and $\varphi$ are concerned with the values of LSDS settling time and dumping ratio.

As the result of the stability of the polynomials $[D(s^*, H_i)]$, $[D(s^{**}, H_i)]$, $i \in \{\overline{0, p}\}$, the satisfaction of the requirements for the system behavior in the states to be considered. The use of results [4] allows to create stability analysis by solving the analogous problem for the group of the polynomials with fixed coefficients. That is why the final part of the design procedure is connected with determination of subsystems parameters $\gamma_1, \gamma_2, \ldots, \gamma_\lambda$ from the totality of nonlinear equations and inequalities.

The method was applied to the design of fault-tolerant LSDS for aircrafts and their engines.

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