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CONSTRUCTION OF A COMPARISON SYSTEM FOR MULTI-DIMENSIONAL CONTROL PROCESSES WITH INTERVAL STATE MATRIX

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A procedure of construction of a comparison system for multi-dimensional control processes with interval state matrix is proposed. The procedure uses modal control methods in a statement of an inverse dynamic problem.

КОНСТРУИРОВАНИЕ СИСТЕМЫ СРАВНЕНИЯ ДЛЯ МНОГОМЕРНЫХ ПРОЦЕССОВ УПРАВЛЕНИЯ С ИНТЕРВАЛЬНОЙ МАТРИЦЕЙ СОСТОЯНИЯ

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Предлагается процедура конструирования системы сравнения для многомерных процессов управления с интервальной матрицей состояния. Процедура использует методы модального управления в постановке обратной задачи динамики.

A problem of a construction of a comparison system for a multi-dimensional control process with an interval state matrix is formulated and solved [1]. A multi-dimensional plant is included in this process. The plant has a linear vector-matrix description

$$\dot{x}(t) = Ax(t) + Bu(t);\ x(0);\ y(t) = Cx(t), \tag{1}$$

where $x(t)$ is a state vector, $u(t)$ is a control vector, $y(t)$ is an exit vector; $x \in R^n$; $u \in R^r$; $y \in R^m$; A, B, C are state, control and exit matrices, respectively, such that $A \in R^{n\times n}$, $B \in R^{n\times r}$, $C \in R^{m\times n}$.

The problem considered is solved under following assumptions.

Assumption 1 (A1). It is supposed that in (1), the basis of representation is choosen such that entries $A_{ij}; i,j=\overline{1,n}$ of the state matrix A of the plant are interval numbers, the matrices B and C are fixed.

For convenience of computations, it is supposed that A is given in the Frobenius basis; then it is a companion matrix with respect to its interval characteristic polynomial

$$\det(\lambda I - A) = \lambda^n + [a]_1\lambda^{n-1} + [a]_2\lambda^{n-2} + \ldots + [a]_n.$$

Assumption 2 (A2). It is supposed that a pair of matrices (A,B) is completely controlled and a pair of matrices (A,C) is completely observable for all $A_{ij} \in [A]_{ij} : i,j=\overline{1,n}$.

The problem is solved on the basis of the comparison system apparatus [4] combined with modal control methods formulated in a "soft form" of the inverse dynamic problem [2,3]. The problem is solved in two steps. On the first step for plant (1) with interval matrix A, a modal model (MM) [3] with state matrix Γ is constructed. The matrix Γ has a given degree of stability η. On the second step for a system with interval state matrix designed by modal control methods, a comparison system is constructed.

Under constructing the matrix Γ, parameters of the modal Gershgoring's covering S_G [5] of the matrix A of the initial plant and a property of an algebraic spectrum of eigenvalues of a matrix-value function of a matrix are used.

Definition 1 (D1). If a domain of a plane of eigenvalues of an arbitrary quadratic matrix is formed by intersection of Gershgoring's circles [5] such that

$$S_G = \{\lambda_G : \ \underline{\mathrm{Re}\,\lambda_G} \le \mathrm{Re}\,\lambda \le \overline{\mathrm{Re}\,\lambda_G}, \quad 0 \le |\,\mathrm{Im}\,\lambda\,| \le \overline{\mathrm{Im}\,\lambda}\},$$

then the domain is called modal Gershgoring's covering S_G.

Evidently, if the matrix A has interval parameters, then the parameters of modal Gershgorin's covering S_G have also interval properties. For solving this problem, let us introduce parameters

$$\overline{\mathrm{Re}\,S_G} = \max_{\substack{A_{ij}\in[A_{ij}]\\ i,j=\overline{1,n}}} \overline{\mathrm{Re}\,\lambda_G}, \quad \underline{\mathrm{Re}\,S_G} = \min_{\substack{A_{ij}\in[A_{ij}]\\ i,j=\overline{1,n}}} \overline{\mathrm{Re}\,\lambda_G},$$

and also realizations $\underline{A}$ and $\overline{A}$ of the interval matrix A. Here $\underline{A}$ represents the realization of the matrix A whose modal Gershgoring's covering has the right bound $\underline{\mathrm{Re}\, S_G}$ and $\overline{A}$ represents the realization of A with the right bound of covering S_G as $\overline{\mathrm{Re}\, S_G}$. Then the matrix Γ of the constructing modal model with stability degree greater or equal to η may be given in the form

$$\Gamma = \overline{A} - (\overline{\mathrm{Re}\, S_G})I - \eta I. \tag{2}$$

As shown in [3], if T is the solution of the Sylvester equation

$$T\Gamma - AT = -BH, \tag{3}$$

with an arbitrary matrix H consistent with B in dimension and forming an observable pair (Γ, H), then the matrix K of feedback can be calculated with the help of the relation $K = HT^{-1}$. Now, if the matrix K of feedback is decomposed into the components K_g and K_x according to the matrix relation

$$[K_g K_x]\begin{bmatrix} C \\ I \end{bmatrix} = K, \tag{4}$$

then the system obtained by the combination of plant (1) and the control law with matrices K_g in a circuit and K_x in a feedback has the following form:

$$\dot{x}(t) = Fx(t) + Gg(t);\ x(0), \tag{5}$$

where $F = A - BK$, $G = BK_g$, $g(t)$ is an external action.

We shall estimate properties of convergent processes in system (5) with an interval state matrix by use a comparison system in the Shilak form (see [4])

$$\dot{z}(t) + \alpha z(t) = \gamma\|g(t)\|, \quad z(0) = \beta\|x(0)\|, \tag{6}$$

which leads to the majorant inequality

$$\|x(t)\| \le \beta e^{-\alpha t}\|x(0)\| \quad \forall t. \tag{7}$$

Estimates of α, β and γ according to [3,4] can be obtained by the following matrix relations

$$F^{\mathsf{T}}P + PF = -Q, \tag{8}$$

where $P = P^{\mathsf{T}}$, $P > 0$, $Q = Q^{\mathsf{T}}$, $Q > 0$,

$$\det(\mu P - Q) = 0, \tag{9}$$

which allow us to write the following relations for α, β and γ :

$$\alpha = \mu_{\min}/2; \quad \beta = (\lambda_{p^{\max}}/\lambda_{p^{\min}})^{1/2} = C\{P\}^{1/2}, \tag{10}$$

$$\gamma = \beta\lambda_{p^{\max}}^{1/2}\alpha_{G^{\max}}, \tag{11}$$

where $C\{P\}$ is a spectrum number of conditionality of the matrix P, $\alpha_{G^{max}}$ is the maximal singular number of the matrix G.

Essentially, the following is proved.

Assertion 1 (AA1). *According to the rule of constucting matrix F by means of modal control, estimates of α and β (10) take interval values $\overline{\alpha}$ and $\underline{\beta}$ on the set of interval values of parameters of the state matrix A of initial plant (1).*

The key point in the obtained relations and, therefore, in values of estimates of α and β is the pair of matrices (P, Q) calculated according to the Lyapunov equation (8). For calculation of matrices P and Q, the following assertion turned out to be useful.

Assertion 2 (AA2). *The matrices P and Q of the form*

$$P = (M^{-1})^{\mathsf{T}}(M^{-1}), \quad Q = -(M^{-1})^{\mathsf{T}}(\Lambda + \Lambda^{\mathsf{T}})(M^{-1}), \tag{12}$$

where the matrices M and Λ satisfy the Sylvester matrix equation

$$M\Lambda - AM = -BL, \tag{13}$$

where (A, B) is a controlled pair, (Λ, L) is an observable pair, and the spectra of eigenvalues of the matrices Λ and A do not intersect, satisfy the Lapunov matrix equation (8).

The proof assertion (AA2) is given in [3].

If the matrix Λ in the Sylvester equation (3) is diagonal (in the case of presence of complex eigenvalues, it is block-diagonal), then the matrix P

of form (12) allow us to control properties of a space of an operator with the matrix F of a system with interval parameters in the form of control of its conditionality.

The complete procedure of the solution of the problem can be written as

1. Construction (A, B, C) of description of plant (1).
2. Definition of the right bound of the modal Gershgoring's covering.
3. Prescribing modal model (2) with the observable pair of matrices (Γ, H).
4. Solving Sylvester equation (3) for T.
5. Synthesis of the matrix of system (4) by the feedback $K = HT^{-1}$, $F = A - BK$ and decomposition its on the components K_g and K_x by the use of relation (4).
6. Construction of the diagonal (block-diagonal) matrix Λ (8).
7. Computation of the diagonalizing matrix M of a similarity transformation constructed on eigenvectors of the matrix F.
8. Construction of the matrices P and Q of form (12).
9. Construction of estimates of α and β of form (10) and γ of form (11) of parameters of comparison system (6) for multidimensional control processes on system (5).

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