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IDENTIFICATION OF NONLINEAR DYNAMIC OBJECTS USING INTERVAL EXPERIMENTAL DATA

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For identification of nonlinear dynamic systems, the Hammerstein and Wiener models have find wide application. Usually in this case deterministic systems or systems with random disturbances are considered. This paper deals with the case where the disturbances acting on the object have nonstatistical nature and are bounded in amplitude, that is, the case of interval experimental data is discussed.

ИДЕНТИФИКАЦИЯ НЕЛИНЕЙНЫХ ДИНАМИЧЕСКИХ ОБЪЕКТОВ ПО ИНТЕРВАЛЬНЫМ ЭКСПЕРИМЕНТАЛЬНЫМ ДАННЫМ

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Для идентификации нелинейных динамических систем широкое применение нашли модели Гаммерштейна и Винера. Как правило, при этом рассматриваются детерминированные системы или системы со случайными возмущениями. В данной статье изучается случай, когда возмущения, действующие на объект, носят нестатистический характер и ограничены по амплитуде, т.е. рассматривается случай интервальных экспериментальных данных.

1. Formulation of the identification problem

Assume that the considered object has the Hammerstein structure (fig. 1) or the Wiener structure (fig. 2) and the linear dynamic part is stable. Measured input and output values are taken simultaneously at times $t = \overline{1, N}$.

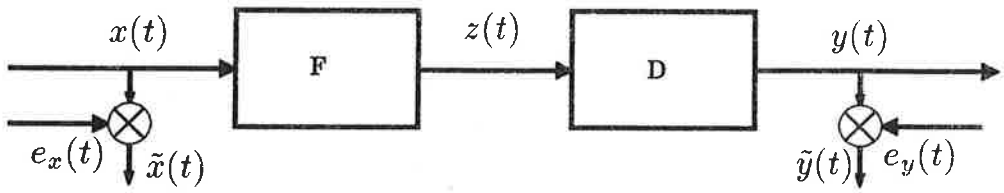

Fig. 1. The Hammerstein model.

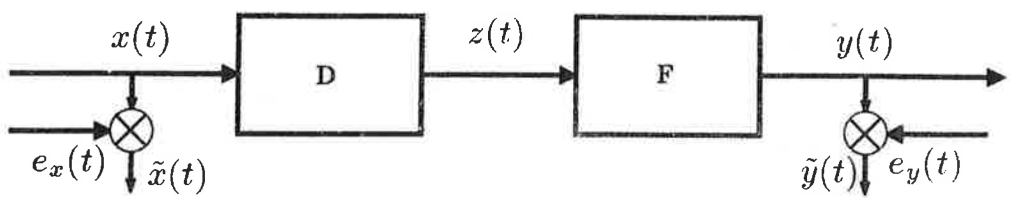

Fig. 2. The Wiener model.

Designations. F is a static non-linearity, D is a linear dynamic part, $x(t)$ and $y(t)$ are input and output variables of the system, $\widetilde{x}(t)$ and $\widetilde{y}(t)$ are measured input and output variables of the system, $z(t)$ is a intermediate non-measured value, $e_x(t)$ and $e_y(t)$ are input and output errors.

Suppose that input and output errors are bounded in amplitude

$$| e_x(t) | \leq \Delta_x(t); \quad | e_y(t) | \leq \Delta_y(t),$$

where $\Delta_x(t), \Delta_y(t)$ are known to the investigator.

Then as a result of an experiment, interval data can be obtained which can be represented as a collection of N intervals:

$$x(t) \in [\widetilde{x}(t) - \Delta_x(t), \widetilde{x}(t) + \Delta_x(t)] = \left[x^-(t), x^+(t)\right], \ t = \overline{1, N}, \tag{1}$$

$$y(t) \in [\widetilde{y}(t) - \Delta_y(t), \widetilde{y}(t) + \Delta_y(t)] = \left[y^-(t), y^+(t)\right], \ t = \overline{1, N}, \tag{2}$$

where $x^-(t), x^+(t), y^-(t), y^+(t)$ are respectively lower and upper boundaries of input and output intervals.

The description of the dynamic part is sought as the linear difference equation and that of the static part — as the polynomial of k-th degree. For Hammerstein model, we write

$$z(t) = F(x(t)) = \sum_{l=1}^{k} c_l x^l(t), \tag{3}$$

$$y(t) = \sum_{i=1}^{n} a_i y(t-i) + \sum_{j=0}^{m} b_j z(t-\tau-j), \tag{4}$$

for Wiener model:

$$z(t) = \sum_{i=1}^{n} a_i z(t-i) + \sum_{j=0}^{m} b_j x(t-\tau-j), \tag{5}$$

$$y(t) = F(z(t)) = \sum_{l=1}^{k} c_l z^l(t), \tag{6}$$

where a_i, b_j, c_l are unknown coefficients, τ is a transport delay of the dynamic element.

The identification problem consists in following: from interval experimental data (1), (2), to estimate the type of the model, orders n, m and k; to find the estimation of model parameters a_i, b_j, c_l and the estimation of the transport delay time τ.

2. Estimation of the transport delay time and choice of the type of the model

The step signal is sent to the object, the input and output of the model are measured. The estimation of the transport delay time is taken to be the time over which the interval output of the object $[y^-(t), y^+(t)]$ includes zero, that is, the hypothesis about the equality to zero of the output object:

$$\hat{\tau} = T,\ T\ :\ \forall t \in [0, T] \Leftrightarrow [y^-(t), y^+(t)] \ni 0$$

can be taken.

For estimation of the type of the model, two step signals with amplitudes A_1 and A_2 are sent to the object and response of the object is fixed: $[y_1^-(t), y_1^+(t)], [y_2^-(t), y_2^+(t)]$, $t = \overline{1,N}$. Since when the signal passed through the nonlinear element the form of the signal is not distorted, then for Hammerstein model the following equation has to be held:

$$\frac{y_1(t)}{y_2(t)} = \text{const}, \quad t = \overline{1,N}. \tag{7}$$

Under conditions of interval uncertainty of data, the relation (7) is verified as the hypothesis about the existence of the model of the form $y_1(t) = \alpha y_2(t)$. When the appropriate set of admissible models is non-empty, the Hammerstein model is proposed. Otherwise, in general it is difficult to say, which model the object can admit (Wiener or a model of more complex type).

We choose the estimates of values n, m and k as the minimal ones at which the appropriate sets of admissible interval models (3) and (4) for the Hammerstein model (or (5) and (6) for the Wiener model) are simultaneously non-empty. In the sequel we shall assume that the values n, m and k are known.

3. Finding interval static model

The step signal with k (or greater) different amplitudes is sent to the object. The duration of every step T_e is required to be sufficiently large for the output signal to be near to the steady value. Without loss of generality we shall assume that the transfer coefficient of the linear dynamic part is equal to 1 (it is accounted in the non-linear element). Then according to the premise about the stability of the linear part, steady values of non-measured variable $z(t)$ (at every step of the input signal) lie in the interval:

for the Hammerstein model

$$z(t) \in \left[y^-(t) - \Delta_\infty,\ y^+(t) + \Delta_\infty\right] = \left[\hat{z}^-(t), \hat{z}^+(t)\right],$$

for the Wiener model

$$z(t) \in \left[x^-(t),\ x^+(t)\right] = \left[\hat{z}^-(t), \hat{z}^+(t)\right],$$

where Δ_∞ is an amplitude of the error of the steady mode

$$| y(t) - y(\infty) | \leq \Delta_\infty, \; t > T, \; T < T_e. \tag{8}$$

From data of input and output variables of the static element for the Hammerstein model

$$\left[x_j^-, x_j^+\right] <------> \left[\hat{z}_j^-, \hat{z}_j^+\right], \quad j = \overline{1,k}$$

(these points are taken at times of the end of every step, when condition (8) is held), by the method of the interval data analysis the set Ω_c of admissible static models is constructed. The set Ω_c is defined by the system of the inequalities

$$\Omega_c = \left\{ \overrightarrow{c} : \min_{x_j \in [x_j^-, x_j^+]} \sum_{l=1}^{k} c_l x_j^l \leq \hat{z}_j^+, \quad \max_{x_j \in [x_j^-, x_j^+]} \sum_{l=1}^{k} c_l x_j^l \geq \hat{z}_j^+, \; j = \overline{1,k} \right\}. \tag{9}$$

Similarly in the case of the Wiener model form the data

$$\left[\hat{z}_j^-, \hat{z}_j^+\right] <------> \left[y_j^- - \Delta_\infty, \; y_j^+ + \Delta_\infty\right], \quad j = \overline{1,k}$$

the set Ω_c is constructed.

4. Finding interval dynamic model

Assume that the set Ω_c is non-empty. Experimental data of transient modes are used for the estimation of the set of parameters Ω_{ab} of the dynamic model. In this case we get a collection of interval data (1), (2). It is necessary to find estimating intervals, which with a guarantee contain true values of a non-measured variable $z(t)$. In the case of the Hammerstein model, static model (3) is used for prediction:

$$\hat{z}^-(t) = \min_{\substack{\overrightarrow{c} \in \Omega_c, \\ x(t) \in [x^-(t), x^+(t)]}} \sum_{l=1}^{k} c_l x^l(t) - \Delta_z(t)$$

$$\hat{z}^+(t) = \max_{\substack{\overrightarrow{c} \in \Omega_c, \\ x(t) \in [x^-(t), x^+(t)]}} \sum_{l=1}^{k} c_l x^l(t) + \Delta_z(t),$$

where $c = (c_1, c_2, \ldots, c_k)$; $\Delta_z(t) = \Delta_y(t) + \Delta_\infty$.

For the Wiener model it is possible to use static model (6) with interval coefficients:

$$y(t) = F(z(t)) = \sum_{l=1}^{k} [c_l^-, c_l^+] z^l(t), \tag{10}$$

where $c_l^- = \min_{\Omega_c} c_l, \qquad c_l^+ = \max_{\Omega_c} c_l.$

Assume that function (6) is one-to-one function in the operating range $[z_{\min}, z_{\max}]$. Let us search the interval $[\hat{z}^-(t), \hat{z}^+(t)]$ and values inside it are roots of the equation

$$[y^-(t), y^+(t)] = [f^-(z(t)), f^+(z(t))],$$

where $f^-(z(t)), f^+(z(t))$ are respectively lower and upper boundaries of the interval function $F(z(t))$ defined by equation (10).

According to rules of the interval arithmetic [4] we obtain that if $z1$ and $z2$ are respectively roots of equations

$$f^-(z(t)) - y^+(t) = 0; \qquad f^+(z(t)) - y^-(t) = 0, \tag{11}$$

then estimations $\hat{z}^-(t)$ and $\hat{z}^+(t)$ are defined as

$$\hat{z}^-(t) = \min(z1, z2); \qquad \hat{z}^+(t) = \max(z1, z2), \quad t = \overline{1, N}.$$

Equations (11) have the form $g(z) = 0$ and can be solved by known methods, such that the secant method,

$$z^{(i+1)} = z^{(i)} - \frac{g(z^{(i)})}{s(z^{(i)})}, \quad \text{where} \quad s(z^{(i)}) = \frac{g(z^{(i)}) - g(z^{(i-1)})}{z^{(i)} - z^{(i-1)}}.$$

From the collection of interval input and output data of dynamic model, for the Hammerstein model,

$$[\hat{z}^-(t), \hat{z}^+(t)] < - - - - - - > [y^-(t), y^+(t)], \quad t = \overline{1, N}$$

the set Ω_{ab} of admissible dynamic models is defined:

$$\Omega_{ab} = \Bigg\{ \vec{a}, \vec{b} : \\ \min_{\substack{z(t)\in[\hat{z}^-(t),\hat{z}^+(t)];\\ y(t)\in[y^-(t),y^+(t)]}} \left(\sum_{i=1}^{n} a_i y(t-i) + \sum_{j=0}^{m} b_j z(t-\tau-j) \right) \le y^+(t) \\ \max_{\substack{z(t)\in[\hat{z}^-(t),\hat{z}^+(t)];\\ y(t)\in[y^-(t),y^+(t)]}} \left(\sum_{i=1}^{n} a_i y(t-i) + \sum_{j=0}^{m} b_j z(t-\tau-j) \right) \ge y^-(t), t = \overline{1,N} \Bigg\}. \tag{12}$$

Similarly we find the description of the dynamic part of the Wiener model.

5. Features of the interval model

As we have seen, the identification problem is reduced to solving the system of inequalities (9) and (12). The sets of solutions Ω_c and Ω_{ab} in general are not convex, but they are convex in every quadrant of a coordinate system. The authors have developed an algorithm of finding an exact description of these sets as a polygon. The algorithm permits to find all active vertices of the polygon and determine the No's of active tests. In the identification procedure, all approaches of the interval analysis method are used: the check of the signification of model coefficients, the check of the adequacy, the choice of optimal models, the evaluation of point and interval estimations of coefficients, the prediction of output values and the estimation of the model efficiency [1], [2], [3].

References

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