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INTERVAL AUTOMATIC SYSTEMS — THEORY, COMPUTER-AIDED DESIGN AND APPLICATIONS

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A concept of interval automatic system (IAS) is introduced, an information about basic steps of the development of the theory of IAS and methods of solving its problems are given. Based on an example of the program package "Analysis and synthesis of interval automatic systems (ASIAS)", the computer implementation of design methods of IAS is considered. Descriptions of technical IAS are given.

ИНТЕРВАЛЬНЫЕ АВТОМАТИЧЕСКИЕ СИСТЕМЫ — ТЕОРИЯ, АВТОМАТИЗИРОВАННОЕ ПРОЕКТИРОВАНИЕ И ПРИЛОЖЕНИЯ

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Вводится понятие интервальной автоматической системы, даются сведения об основных этапах становления теории ИАС и методах решения ее задач. На примере комплекса программ "Анализ и синтез интервальных автоматических систем" (КП АСИАС) рассматривается компьютерная реализация методов проектирования ИАС. Приводятся описания технических ИАС.

1. Theory of interval automatic systems

The analysis of literature on the automatic control theory (ACT) for about 15 last years shows that in the ACT a new direction concerning with investigation of *interval dynamic systems* arises [1], [2], [3]. The systems have *uncertain* parameters with values belonging to intervals with given boundaries. Because of principle impossibility of constructing correct mathematical models, the interval uncertainty (IU) is inherent not only to parameters of any kind od systems (continuous, discrete, linear, nonlinear, distributed, stationary, nonstationary), but also to another characteristics (outer disturbances, initial conditions). Taking into account the specifics of considerations of these systems by IAS, we shall call the systems *interval automatic systems* (IAS). IAS can be divided, respectively, into automatic systems (AS) with the interval uncertainty (IU) of parameters (ASIUP), AS with IU of disturbances (ASIUD), AS with IU of initial conditions (ASIUIC), and also IAS with combined interval uncertainty (ASCIU).

The initial reference point in developing theory of IAS is apparently the paper by S. Faedo [4], where an interval mathematical plant (a polynomial with coefficients as intervals) was first clearly described and a problem of the analysis of plant stability was solved also. Speaking in modern language, an interval version of the Rauth stability criterion was obtained in this paper. However, the wide front of IAS has been formed only after the paper by V. L. Kharitonov [5] had appeared.

Solving usual design problems for IAS is faces with considerable difficucties. If we do not accent an attention on the character of the parametric uncertainty, then for their overcoming, we can apply methods of the *robustness* theory. However, compared to the robustness theory that has norms as a basic mathematical instrument, the theory of IAS accounting the intervality allows us to obtain more practical solutions on the basis of the method of *corner elements* (CE). The method uses specific points of intervals (boundary, center, and so on) and in fact refers to showing problems and plants for which the proof of a presence of some property at these points allows us to guarantee the existence of this property also in a whole set of remaining points of an interval plant description. The problem of the stability analysis of an interval polynomial is an effective example of obtaining good results of the method of CE. This problem have been solved in the mentioned already paper [5] (because of this, the method of corner elements is called the Kharitonov one).

The great majority of investigations in the field of IAS is now carried by the method of CE. However evidently, possibilities of the method are limited and for interval problems, it is necessary to search specific mathematical tools. This conclusion is confirmed particularly by difficulties of an application of CE method to discrete IAS [2].

The first attempt of founding a special "interval" mathematics and applying it in problems of ACT in our country was undertaken in 1980 [6]. It was the *algebra of uncertainties* developed by the author and supplemented then by the allocation operation (nonstandard interval subtraction) and by rules of solving systems of linear interval algebraic equations. In the sequel, it has been found that these results (obtained independently) coincide with fundamental axioms of the *interval analysis* (IA) [7]. Approaches of IA have been applied (1981–1984) to basic problems of the linear theory of IAS — the analysis of stability [8], controllability [9], quality [10], the synthesis of regulators (by the modal control method [11] and analytic design [12]). The obtained results are encouraging since IA allows us to perform *localizational computations* of interval reference data, that is to obtain the shell of a correct solution. Then first attempts of applying IA in adaptive control problems [13] and optimization methods [14] were undertaken. Along with the merits of IA, the basic deficiency of this method had been exposed. The deficiency consists in the trend to increase the sizes of the mentioned earlier shell ("blowing"). At present, one carries out an application of IA to nonlinear problems of automatic control with regard to nonstationarity, discretness, and distributive nature of plants.

It should be noted that the algorithm complexity of solving automatic control problems requires the consideration of computational errors (particularly for plants of high dimension) and here there is also wide field for using IA.

The foregoing permits the method IA to be thought as a basic method of IAS investigations.

2. Computer implementation of the theory of IAS

It is likely paper [15] should be regarded in our country as the first attempt of solving problems of the theory of IAS on ESM using interval analysis. Interval computations are performed by direct programming of intervally-arithmetic operations and by choice of the largest and smallest numbers from four obtained numbers using functions AMIN1 and AMAX1 of FORTRAN-4, and computational errors do not be taken into account. For removal effect of "blowing" of the interval shell, nonpopular algorithms are programmed (for example, the method of direct expansion of a characteristic polynomial). Since the work with these algorithms is very laborious, much attention was given to the minimization of computational cost (it is interesting that allows one to obtain important theoretical results noted in [3]). The struggle with the "blowing" and an expierence of solving practical problems [16] lead to a thought about necessity of use and reasonable combining *symbolic* and *interval* computations. The expierence was taken into account under creating the program package ASIAS, the structure of a educational version of which is shown in fig. 1.

In the educational version of the program package ASIAS, the problems of analysis and synthesis of linear continuous automatic systems with an interval uncertainty of parameters (ICASIUP) are solved at present. Transfer functions and systems of first order linear differential equations are mathematical models of LCASIUP. The plants may be *pure interval* and *functionally interval* ones. If in the description of a plant different interval values enter one time and in the first degree, then the plant is called *pure interval* one. The description of functionally interval plant contains arbitrary functions of interval uncertain values. Functional dependence may be linear or nonlinear (polylinear, signumial).

The first step in an analysis of LCASIUP is the definition of controlling its plant with the help of the program XAUR1 in which the interval version of the Kalman criterion is implemented. The second step is the definition of the stability of LCASIUP which is carried out by algebraic and frequency methods. If a system has a model in the form of a transfer function (TF), then the Kharitonov criterion is applied to the interval polynomial of its denominator.

The stability of an interval transfer function can be defined also by majorants and minorants of the domain of possible values of the following frequence characteristics: amplitude, phase, real, imaginary ones. If the LCASIUP is a plant of the Cauchy type closed by feedback in complete state vector, then for an analysis of the stability, criteria of the stability of interval matrices are used.

The analysis of quality indices of LCASIUP can be performed from the location of roots of an interval characteristic polynomial of a closed system (ICPCS) in a simple connected and multiconnected domains on the complex plane, from frequency characteristics, transient processes, that is, responses to standard input signals and nonzero initial conditions.

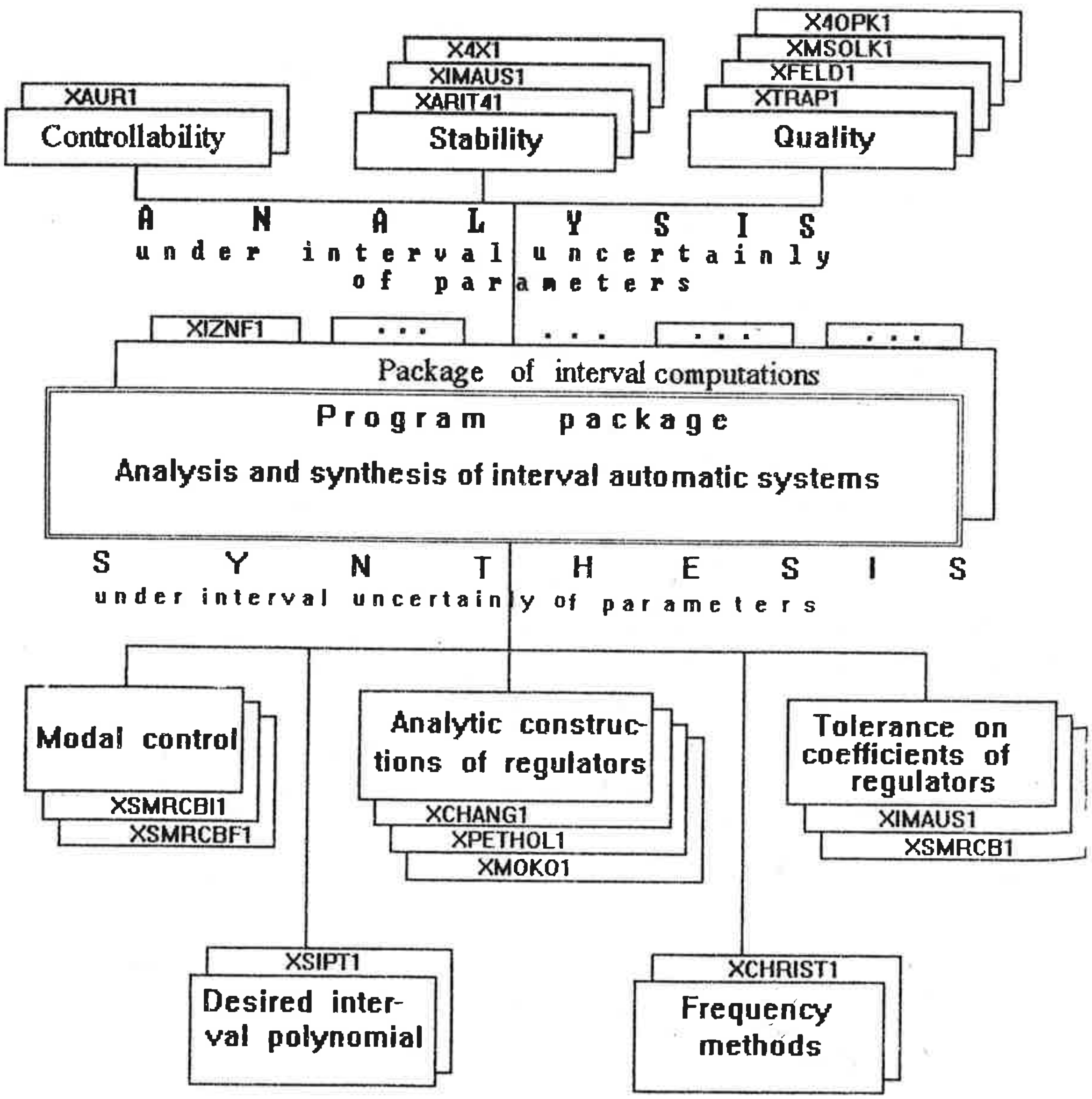
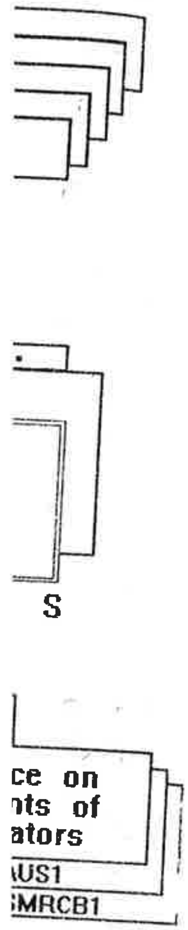


Fig. 1. Structure of an educational version
of the program package ASIAS.



Synthesis of regulators is implemented by interval versions (IV) of modal control methods (MCM), analytical design (AD), and frequency characteristics (FC). In the IVMCM, the way of including interval characteristic polynomial of a closed system (ICPCS) into a desired (preliminary synthesized) ICPCS in used [11].

A distinguishing feature of the program package ASIAS is that the operations with interval values are carried out according to the rules of the interval analysis. Therefore, the package of interval computations is one of constituents of its software [17]. The package works with following interval mathematical objects: functions, systems of linear algebraic equations, matrices, differential equations.

For interval functions, the package allows one to find an interval of values, to construct boundary functions (the majorant and minorant); for plynomials with pure and functionally interval coefficients, the domains of root localization (simpleconnected and multiconnected) are found. For systems of linear interval algebraic equations, different kinds of solutions (unified, admissible, guaranteed, and so on) are defined. The matrix algebra is represented by an expansion of a characteristic polynomial, a construction of an adjoint matrix, a multiplication, an involution, a computation of an interval of determinant values. For interval differential equations, the domains (tubes) of solutions are constructed.

The programs realized the most typical methods are included in the educational version of PP ASIAS. The industrial version in this sense is richer; besides, symbolic computations and the language PASCAL-XSC are used in [18]. Both versions are destined for IBM PC AT/XT, MS-DOS 3.0 and higher.

3. Experience of practical use of the theory of IAS

During several years, PP ASIAS is used in the educational process in Saratov Polytechnic Institute. The industrial application represents the solving of problems of the synthesis of stabilization of regulators of different helicopter systems: a helicopter moving in order [6], [8], [11], [19], a helicopter towing complex [16], [20], a helicopter-crane [21].

The helicopter towing complex (HTC) is intented for the transportation of towing objects (TO) of different assignment along water surface (fig. 2). The complex consists of helicopter, towing rope, and TO. In papers [16], [20], the lateral motion of HTC in the coordinate plane X_gOZ_g is considered; O_1 is the center of gravity of the helicopter, O_2 is the mounting point of the rope which is defined by values a and b_r. The deviations $\Delta Z_g h$ of the helicopter and $\Delta Z_g t$ of the TO from the given trajectory of motion are controlled. The dependence of the towing regime on a series of uncertain parameters is the characteristic property of that regime. These parameters are: draft force under towing F, balancing value of the pitch angle v_0, inclination angle of the rope to the horisontal plane δ_0, length of the towing rope L. The value of uncertainty may be as great as 400%. The work of the HTC is described by a system of five differential equations with nonlinear dependence of coefficients on the parameters mentioned above. By the use of programs entered into the industrial version (IV) of the PP ASIAS, the regulator of stabilization of the TO on the given trajectory of the motion has been synthesized.

Helicopters have also a wide application as a flight cranes. A weight is suspended to a helicopter by a rope and transported to the assignment place. During the drag of the helicopter, vibrations of the weight arise; the vibrations are a severe problem under performing works of the weight mounting. Damping of the vibrations take place during a long time since the aerodynamic resistance to the weight motion at low velocities is small. Therefore, before the weight mounting it is required to damp quickly the vibrations of the weight simultaneously with the stabilization above the hanging point. Complexity of resolving this problem for a flier (for example, in conditions of bad visibility) is responsible for the necessity of the development of an automatic system.

In [21] for a hanging regime, a regulator with constant coefficients having tolerances was calculated using IV PP ASIAS. The calculation was carried out for small lengthes of the rope based on uncertainty of the mass of the weight and some constructive parameters of the helicopter starting from desired indices of quality of stabilization processes above the hanging point and these indices of damping of the weight vibrations. The rope length was also considered as the intervally uncertain parameter. The total number of uncertain parameters in the plant is equal to 3, and the rope length can vary 2 times, the mass of the weight can vary 5 times, remaining parameters are known to 20% accuracy. The uncertain parameters enter nonlinearly in a mathematical model of 6 differential equations.

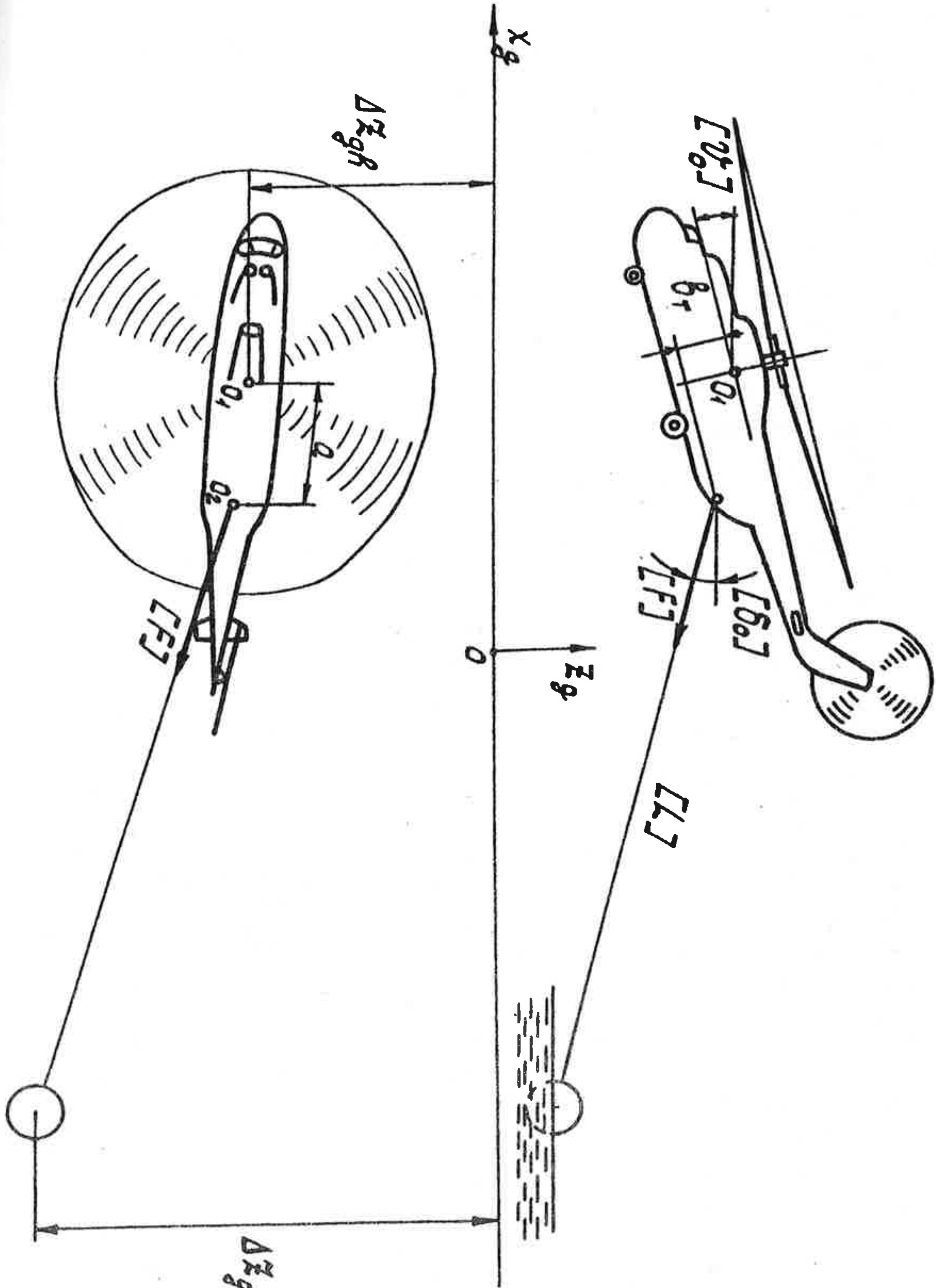


Fig. 2. Helicopter towing complex

At present PP ASIAS is used in the educational process of Electrostal branch of Moscow Institute of Steel and Alloys and in the developing of high-precise drives of metal-cutting benches.

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