An Overview of the Upcoming IEEE P-1788 Working Group Document: Standard for Interval Arithmetic

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Outline

Introduction

Issues with Interval Standardization

Current Status

Resources

Why Standardize?

General Issues

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Predictability: Is the behaviour of a system deducible from well-publicized documentation?

Reproducibility: Are the results from one implementation of the standard on any platform the same as the results on any other platform? (desirable for debugging and certification purposes)

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- Numerous interval arithmetic packages have been developed. The results of interval operations differ from package to package. Re-use of these packages by developers wanting interval arithmetic is hampered by lack of predictability.
- Lack of a central standard leads to more error-prone development when using an interval package, negating the reliability and mathematical rigor inherent in interval operations.

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5. However, certain details need to be decided.

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  2. require the lower bound to be the closest floating point number ≤ 0 and the upper bound to be the closest floating point number ≥ 5,
  3. or do we require (or allow) something else?
- If the IEEE 754-2008 floating point standard is assumed, it is not hard to develop systems satisfying requirement 2.

Elementary functions
(Issues needing decision)

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- Which library functions should be included?
- What (if any) accuracy should be required?

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5. The standard should accommodate both 3 and 4.

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- What about alternative representations, such as midpoint-radius (nominal value with an error tolerance)?

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- Should a loss-free interchange format be standardized, to facilitate loss-free interchange of interval data between conforming implementations?

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- Level of specification of representations (bit level, or some higher functional level)
- Inclusion of the exact dot product and an associated long data type versus a correctly rounded dot product versus something else.
- Other minor issues.

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- The process needs to be completed by December, 2014.

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- Once registered with IEEE, access details for the mailing list and standing document are available from me (`rbk@louisiana.edu`).