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On Utilizing Disconnected Images within GlobSol's Constraint Propagation Software

Ralph Baker Kearfott

Department of Mathematics
University of Louisiana at Lafayette

SCAN 2010

Discontinuous Images in Constraint Propagation

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General Context – Constraint propagation

Components of Images

- Constraint propagation is commonly used as one of various tools to reduce the size of search regions in branch and bound-based software for global optimization.

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- Constraint propagation is commonly used as one of various tools to reduce the size of search regions in branch and bound-based software for global optimization.
- Constraint propagation greatly speeds the overall solution process and makes some problems that otherwise may not be practical to solve.

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Components of Images

- Constraint propagation is commonly used as one of various tools to reduce the size of search regions in branch and bound-based software for global optimization.
- Constraint propagation greatly speeds the overall solution process and makes some problems that otherwise may not be practical to solve.
- The volume of regions returned from constraint propagation is reduced more if inverse images with multiple, disconnected components are taken, rather than by taking the interval hull of such components and continuing with a single interval.

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Discontinuous Images

An Example

Take the constraint

$$1/x - y = 0$$

along with the bounds $x \in [-3, 3]$ and $y \in [-1, 1]$, solving for x in terms of y gives

$$x \in \{(-\infty, -1] \cup [1, \infty)\} \cap [-3, 3] = [-3, -1] \cup [1, 3].$$

- Similar disconnected solution sets occur when:

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- Similar disconnected solution sets occur when:
  - taking inverses of even-order integer powers,
  - taking inverses of trigonometric functions.
- Should we consider $[-3, -1]$ and $[1, 3]$ separately, or should we simply use $[-3, 3]$ in subsequent calculations?

Individual Image Components

Disadvantages – Should they be used?

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- A disadvantage of proceeding with each component separately is that more boxes are produced.

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- Knowing whether or not to use separate components can only be determined by experimentation within particular software.

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- Knowing whether or not to use separate components can only be determined by experimentation within particular software.
- Here, we report on the effect of using disconnected components with our GlobSol global optimization software.
- We give overall ideas and results here, with more detailed explanations available upon request.

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The Optimization Problem

(Notation)

- We pose the global optimization problem as

$$\begin{array}{l} \text{minimize } \varphi(x) \\ \text{subject to } c_i(x) = 0,\ i = 1, \ldots, m_1, \\ \qquad\qquad\quad g_i(x) \le 0,\ i = 1, \ldots, m_2, \\ \text{where } \varphi : \boldsymbol{x} \to \mathbb{R},\ c_i, g_i : \boldsymbol{x} \to \mathbb{R}, \text{ and} \\ \boldsymbol{x} \subset \mathbb{R}^n \text{ is the hyperrectangle (box) defined by} \\ \qquad\qquad\quad \underline{x}_i \le x_i \le \overline{x}_i,\ 1 \le i \le n, \\ \text{where the } \underline{x}_i \text{ and } \overline{x}_i \text{ are constant bounds.} \end{array} \tag{1}$$

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- Constraint propagation can include φ (using known upper bounds on the global optimum in inequalities), the c_i, and the g_i, as well as subexpressions or individual operations in evaluation of these functions (handled by introducing intermediate variables, analogous to slack variables).

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GlobSol's Overall Algorithm

(An informal summary of the main loop)

1. Remove a box $\boldsymbol{x}$ from a list $\mathcal{L}$ of boxes to be processed.

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2. if $\boldsymbol{x}$ is already sufficiently small, do final analysis and storage of $\boldsymbol{x}$, then remove another box.
3. Use constraint propagation to possibly narrow the coordinates of $\boldsymbol{x}$ or even discard $\boldsymbol{x}$.

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6. Do step 2 if the coordinate widths of $\boldsymbol{x}$ are now sufficiently narrow.

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6. Do step 2 if the coordinate widths of $\boldsymbol{x}$ are now sufficiently narrow.
7. Bisect a selected coordinate of $\boldsymbol{x}$, placing both resulting boxes on the list $\mathcal{L}$.

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Variants of Constraint Propagation and Linear Relaxations

- Both constraint propagation and linear relaxation can be done by

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Variants of Constraint Propagation and Linear Relaxations

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  - solving the compound expressions for φ,c_i, and g_i as the user presents them,

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Variants of Constraint Propagation and Linear Relaxations

- Both constraint propagation and linear relaxation can be done by
  - solving the compound expressions for φ,c_i, and g_i as the user presents them,
  - automatically decomposing the expressions into atomic operations (as a compiler or operator overloading would do), forming an associated family of equations, and inverting (or linearizing) these individual equations,

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- GlobSol presently uses atomic operations both in constraint propagation and in the linear relaxations.

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  - some alternative method.
- GlobSol presently uses atomic operations both in constraint propagation and in the linear relaxations.
- Among systems that use atomic operations, differences in performance occur depending on how the expressions are decomposed and which operations are considered to be atomic.

GlobSol's Constraint Propagation Algorithm

(An informal summary)

1. Compute initial bounds for all variables by evaluating φ, c, and g with interval arithmetic.

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GlobSol's Constraint Propagation Algorithm

(An informal summary)

1. Compute initial bounds for all variables by evaluating φ, c, and g with interval arithmetic.
2. Determine if any of the variables x_ν in the final computation to obtain φ can be narrowed through the condition $\varphi \leq \overline{\varphi}$, where $\overline{\varphi}$ is a known upper bound on the global optimum. Do a similar determination for the conditions $c_i = 0$ and $g_i \leq 0$.

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3. For each variable x_ν was narrowed in step 2, repeat step 2 with the conditions on $\varphi \le \overline{\varphi}$, etc., replaced by each equation containing each x_ν.

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- There is a tuning parameter: A tolerance determining when an $\boldsymbol{x}_\nu$ is narrowed enough to count as narrowed.

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Implementation of Constraint Propagation with Disconnected Intervals in GlobSol

- Often, the constraint propagation results in narrower bounds on an intermediate variable, but not on one of the original variables x_i, $1 \le i \le n$.

[1]Experiments of many years ago indicated storing intermediate variable bounds in lists of unfathomed boxes to be impractical.

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- Often, the constraint propagation results in narrower bounds on an intermediate variable, but not on one of the original variables x_i, $1 \le i \le n$.
- Within GlobSol's overall algorithm, only the bounds $\boldsymbol{x}_i$, $1 \le i \le n$ are stored with unfathomed boxes, and not the bounds on intermediate variables[1].

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Implementation of Constraint Propagation with Disconnected Intervals in GlobSol

- Often, the constraint propagation results in narrower bounds on an intermediate variable, but not on one of the original variables x_i, $1 \le i \le n$.
- Within GlobSol's overall algorithm, only the bounds $\boldsymbol{x}_i$, $1 \le i \le n$ are stored with unfathomed boxes, and not the bounds on intermediate variables[1].
- In GlobSol, we store each disconnected components of particular intervals in the constraint propagation in a special list of intervals to be processed, if the gap is sufficiently large, and we take the interval hull, as before, if the gap is small.

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- When the entire process is finished, we only return original coordinate boxes $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$, and only more than one box if one of these original coordinates was split.

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Disconnected Intervals in GlobSol

(Continued)

- When the entire process is finished, we only return original coordinate boxes $\boldsymbol{x} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$, and only more than one box if one of these original coordinates was split.
- We presently don't do any symbolic preprocessing.

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The Test Problems

We chose only problems from a more extensive test set where we observed differences between our two variants of constraint propagation —

A maintenance optimization model (Claudio Rocco, 1999): Integrality conditions are handled as trigonometric constraints. Disconnected intervals in constraint propagation should separate the intervals.

[2] although this was found to be a bug in GlobSol

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cragg4: Laurent Granvilliers reported better results than GlobSol[2], where Laurent uses extended arithmetic in his constraint propagation.

gcd (from the original GlobSolintegration tests): an easy problem, an unconstrained positive-definite quadratic in 8 variables.

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The Test Problems

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Lagos (an electromagnetic coupling problem): Although $n = 3$, the objective has numerous square roots, sines, cosines, and zero-containing denominators.

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levy3 (from the original GlobSol integration test set): a product of sines and cosines.

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maxdicut (as formulated by Uri Zwick): A rational objective, as well as constraints containing sines, cosines, and arc-cosines of the independent variables.

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pdnlp6i (nonlinear programming problem 3 from the classic Floudas / Pardalos test set): two independent variables and two degree-4 polynomial constraints.

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Prob.	Number of nodes		CPU time	
	Without	With	Without	With
maint. opt.	51	1856	5.13	1051.20
cragg4	1043	162	4.70	0.94
gcd	24	20	0.08	0.07
Lagos	95444	101128	1050.93	1176.20
levy3	696	745	3.65	458.95
maxdicut	246	206	2.52	2.12
pdnlp6i	312	310	0.37	0.35

Here, “Without” means always taking the interval hull, and “With” means following each branch in a disconnected image.

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- There are many variations of the constraint propagation, and each variation's benefit depends on the overall branch and bound process and the efficacy of other elements (such as the linear relaxations). Significantly different results may be obtained depending on these considerations.

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- A copy of this presentation will be posted on `http://interval.louisiana.edu/preprints.html` after this conference.