

Errata and Opinion: “An Interval Entropy Penalty Method for Nonlinear Global Optimization,” by Zhenyu Huang, *Reliable Computing* **4**, pp. 15–25 (1998)

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There is an error in this paper which should be corrected, and there is an ambiguous point which should also perhaps be clarified.

1. The Error

On the top of page 17, it is stated that:

$$\begin{aligned} & \max \{f_1(x), \dots, f_m(x)\} + \max \{g_1(x), \dots, g_m(x)\} \\ & = \max \{f_1(x) + g_1(x), \dots, f_m(x) + g_m(x)\}. \end{aligned}$$

This is well-known to not be true. For example, take $m = 1$, take $f_1(x) = -x^2$, and take $f_2(x) = -(x - 1)^2$. Then $\max\{f\}$ occurs at $x = 0$, and is equal to 0, while $\max\{g\} = 0$ and occurs at $x = 1$. In contrast $\max\{f + g\} = -1/2$, and it occurs at $x = 1/2$. The problem is related to the classical “interval dependency” problem in interval arithmetic.

The correct statement is:

$$\begin{aligned} & \max \{f_1(x), \dots, f_m(x)\} + \max \{g_1(x), \dots, g_m(x)\} \\ & \leq \max \{f_1(x) + g_1(x), \dots, f_m(x) + g_m(x)\}. \end{aligned}$$

The subsequent statement in the paper, i.e., “Thus, the resulting optimization problem is equivalent to a *maxmax* problem,” no longer follows.

2. A Clarification

The entropy function advocated by the author does indeed rigorously bound the original non-smooth objective, as is seen in Lemma 2.1 of the paper, a lemma that appears to be correct. However, the author advocates also using a penalty method to handle constrained *maxmax* problems, by using interval methods to optimize the penalty function. It is unclear to this reader from the presentation in the paper how (and if) the optimum and optimizer of the penalty function bound the optimum and optimizers of the original constrained problem, although

it is known that the solution of the penalty problem tends to the solution of the original constrained problem in the limit. This issue was not addressed in the paper. Thus, all the author seems to have accomplished by rigorously solving the penalty problem is to have obtained rigorous bounds on the solution of an approximation to the original constrained problem.

This difficulty could perhaps be fixed by either stating analytic bounds on the approximation error in the penalty function or by somehow using interval values in the penalty parameter (including ∞ somehow).