1. $\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(\sin(2t)) - \mathcal{L}(u_{2\pi}(t)\sin(t-2\pi))$

$$s^2Y - sy(0) - y'(0) + Y = \frac{2}{s^2+4} - \frac{e^{-2\pi s}2}{s^2+4}$$

$$(s^2+1)Y = \frac{2}{s^2+4}(1-e^{-2\pi s}) + 1$$

$$Y = \frac{2}{(s^2+4)(s^2+1)}(1-e^{-2\pi s}) + \frac{1}{s^2+1}$$

$$\frac{2}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$$2 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$

$$2 = (A+C)s^3 + (B+D)(s^2) + (A+4C)s + (B+4D)$$

$\left.\begin{array}{l} A+C=0 \\ A+4C=0 \end{array}\right\} \Rightarrow A=C=0.$ $\left.\begin{array}{l} B+D=0 \\ B+4D=2 \end{array}\right\} \Rightarrow 3D=2 \Rightarrow \boxed{D=2/3} = B = -2/3.$

Thus $\frac{2}{(s^2+4)(s^2+1)} = \frac{-2}{3}\left(\frac{1}{s^2+4}\right) + \frac{2}{3}\left(\frac{1}{s^2+1}\right) = \frac{-2}{6}\left(\frac{2}{s^2+4}\right) + \frac{2}{3}\left(\frac{1}{s^2+1}\right).$

Thus, $\mathcal{L}^{-1}\left(\frac{2}{(s^2+4)(s^2+1)}\right) = \frac{-1}{3}\sin(2t) + \frac{2}{3}\sin(t).$

Thus, $\boxed{y(t) = \frac{-1}{3}\sin(2t) + \frac{2}{3}\sin(t) - u_{2\pi}(t)\left[\frac{-1}{3}\sin(2t) + \frac{2}{3}\sin(t)\right] + \sin(t)}$