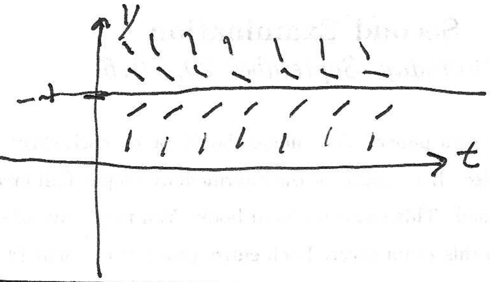
1 (a) The equilibrium is where $y' = \frac{1}{2} - \frac{1}{2}y = 0$, namely, at $y = 1$.

(b) The equilibrium is stable, since $y' > 0$ for $y < 1$ and $y' < 0$ for $y > 1$.

(c)



2 An integrating factor is $e^{\int \frac{2}{t}dt} = e^{2\ln(t)} = t^2$, with which we have: $(t^2 y)' = t^3 \Rightarrow t^2 y = \frac{t^4}{4} + C \Rightarrow$

$y(t) = \frac{t^2}{4} + \frac{C}{t^2}$. $y(1) = 1 \Rightarrow 1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$,

so the solution is: $\boxed{y(t) = \frac{1}{4}t^2 + \frac{3}{4}\left(\frac{1}{t^2}\right)}$

3 The characteristic equation is $r^2 + 2r + 5 = 0$, with solutions $r = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$.

Thus, the general solution can be written as:

$y(t) = C_1 e^{-t}\cos(2t) + C_2 e^{-t}\sin(2t)$

$y(0) = 1 \Rightarrow \boxed{C_1 = 1}$.

$y'(t) = C_1[-e^{-t}\cos(2t) - 2e^{-t}\sin(2t)]$

$\quad + C_2[-e^{-t}\sin(2t) + 2e^{-t}\cos(2t)] = 0$

$\Rightarrow -C_1 + 2C_2 = 0 \Rightarrow \boxed{C_2 = \frac{1}{2}}$

Thus, $\boxed{y(t) = e^{-t}\cos(2t) + \frac{1}{2}e^{-t}\sin(2t)}$

(4) In standard form, the equation is $y''+6y'=-9$, giving a characteristic equation:

$$r^2+6r=0 \Rightarrow r(r+6)=0,$$ so ~~a~~ the general solution to the homogeneous equation is:

$$y_h = C_1 + C_2 e^{-6t}.$$

Since constants are solutions to the ~~non~~ homogeneous equation, a particular solution to the non-homogeneous equation is of the form At. Plugging this into the equation gives:

$$6A = 9 \Rightarrow A = -\tfrac{3}{2}.$$ Thus, the general solution to the full equation is:

$$y(t) = -\tfrac{3}{2}t + C_1 + C_2 e^{-6t}.$$

$$y(0) = C_1 + C_2 = 0.$$

$$y'(t) = -\tfrac{3}{2} - 6C_2 e^{-6t}, \text{ so } y'(0) = -\tfrac{3}{2} - 6C_2 = 1 \Rightarrow -6C_2 = \tfrac{5}{2}$$

$$\Rightarrow C_2 = -\tfrac{5}{12} \Rightarrow C_1 = \tfrac{5}{12}.$$

Thus, $\boxed{y(t) = -\tfrac{3}{2}t + \tfrac{5}{12} - \tfrac{5}{12}e^{-6t}}$