

First Exam

Friday, January 30, 2009

This exam is closed book. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner. Each entire problem is worth 32 points, and 4 points are “free.”

1. Classify each of the following differential equations as linear or nonlinear. In each case, say why.

(a) $\frac{d^2y}{dt^2} + e^t y = t.$

(b) $y' + te^y = 0.$

(c) $y''' + 3(y')^2 + y = e^{-t}.$

(d) $y' \sin(t) + y = e^{-t}.$

2. State the order of each of the equations in Problem 1.

3. Consider

$$\frac{dy}{dt} = 2y - 12.$$

- (a) Sketch a direction field.
- (b) Solve the differential equation, and discuss the relationship between the solution and the direction field you have sketched.
- (c) Solve the associated initial value problem with initial condition $y(0) = 1$.
- (d) Write down an equilibrium solution for y , and state whether that is stable, unstable, or neither. Give a reason for your conclusion.