Math. 302-01
Spring, 2005
R. B. Kearfott

Final Examination

Monday, May 2, 2005, 10:15AM to 12:45PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 16 points, and 4 points are "free."

1. A person swimming in the ocean, one mile south of the beach, encounters a cross-current of 2 miles per hour flowing west, parallel to the beach. How fast and in what direction would the person need to swim to follow a line back to the beach that is perpendicular to the beach, such that the person gets back to the beach in an hour? Give the direction the person must swim as an angle with respect to east. *Hint: It may help to draw a picture.*

2. An object is traveling along a trajectory such that, at time t seconds after starting, it is at $x(t) = t^2$, $y(t) = t^2$, $z(t) = t^{3/2}$ meters from its starting position. On the other hand, the temperature at point (x, y) from the object's starting position is somehow known to be $T(x, y, z) = 25 - .0001x - .0002y - .005z$ degrees centigrade.

   (a) Find the temperature at the object's position one minute after the object has started.

   (b) Find the rate of change of temperature at the object's position one minute after the object has started,

      i. by directly plugging in, and

      ii. by using the chain rule for functions of several variables.

3. An industrial container in the shape of a box (with rectangular sides, intersecting at right angles) is to be constructed. The volume of the box must be 10 cubic meters. The cost of the bottom of the box is \$10 per square meters, the cost of the front and back is \$4 per square meter, the cost of the right and left sides is \$2 per square meter, and the cost of the top is \$1 per square meter. What should the dimensions of the box be to minimize the cost?

4. Use Green's Theorem to evaluate the work needed to traverse the unit circle centered at the origin in a counterclockwise direction, through a vector field given by $\vec{F}(x, y) = (0, x + y)$.

5. Compute the flux of the vector field $\vec{F}(x, y, z) = (x, y, z)$ through that portion of the paraboloid of revolution $z = x^2 + y^2$ for $-1 \le x \le 1$, $-1 \le y \le 1$; assume that the paraboloid surface is oriented outward from the z-axis.

6. Use the divergence theorem to compute the flux through the sphere $x^2 + y^2 + z^2 = 1$ of the vector field $\vec{F}(x, y, z) = (y^2/2, -xy + y, xz - yz)$.