(1) S is flat and $\vec{F}$ is constant, so the flux across S is: $\vec{F}\circ\vec{n}\cdot(\text{Area of } S) = (1,2,3)\circ(0,0,1)\cdot 1 = \boxed{3}$.

---

(2) We use spherical coordinates to compute the surface integral. We have $\rho = 1$, $0 \le \theta \le 2\pi$, $0 \le \varphi \le \pi$,

so the flux integral is:

$$\iint_S \vec{F}\circ d\vec{A} = \int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi} \frac{1}{\rho}(\cancel{\rho\sin\varphi}\,x, y, z)\circ\frac{(x,y,z)}{\rho}\,\rho^2\sin\varphi\, d\varphi\, d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi}\frac{1}{\rho^2}\rho^2\rho^2\sin\varphi\, d\varphi\, d\theta = \int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi}\sin\varphi\, d\varphi\, d\theta$$

$$= \int_{\theta=0}^{2\pi} 2\, d\theta = \boxed{4\pi}.$$

---

(3) We will use the divergence theorem.

$$\operatorname{div}(\vec{F}) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = \sin y^2 + 1 + 0 - \sin y^2 = 1,$$

so $$\iint_S \vec{F}\circ d\vec{A} = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} 1\, dz\, dy\, dx = \boxed{1}$$
