Math. 301-02
Summer, 2005
R. B. Kearfott

Fourth Examination

Monday, July 25, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 25 points.

1. Compute the Taylor polynomial of degree 5 for $f(x) = \cos(x)$, expanded about $x = \pi/2$.

2. (a) Write down the degree 4 Taylor polynomial for $f(x)$, centered at $x = 0$, where
$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$
   (b) Is this Taylor polynomial an overestimate or underestimate for f, for $-\pi/2 \le x \le \pi/2$?

3. What degree Taylor polynomial should be used to ensure that $f(x)$ is approximated with an error of at most 10^{-10} for $|x| \le 1$, for f as in Problem 2?

4. Compute the radius of convergence of the Taylor series for
$$f(x) = \ln(1 + 3x),$$
expanded about $x = 0$.