Math. 301-02
Summer, 2005
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Third Examination
Wednesday, July 13, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 16 points, and 4 points are free.

1. A cylindrical tank with height 30 feet and radius 10 ft sits on a tower so that the bottom of the tank is 75 feet above the ground. Suppose that water is pumped from the ground to fill the tank. How much work, in ft.-lb., does it take to fill the tank? (Water weighs about 62.4 pounds per cubic foot.)
2. Suppose price at which q widgets can be sold is $c(q) = 1500 - 0.1(\sqrt{q})$, whereas the price at which producers are willing to sell q widgets is $p(q) = 220 + 0.1\sqrt{q}$.
   (a) Find the equilibrium quantity and equilibrium price.
   (b) Compute the consumer surplus and the producer surplus.
3. Suppose a probability density of a non-negative quantity x is given by $p(x) = \frac{1}{4}e^{-.25x}$ for $x \geq 0$ and $p(x) = 0$ for $x < 0$.
   (a) Write down an expression for the cumulative probability $P(x)$.
   (b) What is the probability that the quantity is between 1 and 2?
   (c) What is the mean?
   (d) What is the median?
4. Suppose you put 100 at the beginning of each month into a savings account that earns 0.25% interest per month, paid at the end of the month.
   (a) Using the "$\sum$" notation, write down the amount you will have in the account after the beginning of month $n+1$ (starting with $n = 0$ for the initial month).
   (b) Write down this amount in closed form.
   (c) How much will you have after 5 years?
5. Does
   $$\sum_{i=0}^{\infty} \frac{(-1)^i}{i!}$$
   converge? State why.
   Note: $i!$ is the product of the first i integers, where $0!$ is defined to be 1.
6. Compute the radius of convergence of each of the following.
   $$\text{(a)} \quad \sum_{i=0}^{\infty} \frac{x^i}{i!} \qquad \text{(b)} \quad \sum_{i=1}^{\infty} \frac{(-1)^{i-1}}{i} x^i$$