Math. 301-02
Summer, 2005
R. B. Kearfott

Final Examination

Saturday, July 30, 2005, 7:30AM–10:00AM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each entire problem is worth 16 points, and 4 points are "free."

1. Compute the following integrals. Show all work or explain.

$$\text{(a)} \int_{x=0}^{1} \frac{\arctan(x)}{x^2+1} dx \qquad \text{(b)} \int_{x=0}^{\pi/2} x\sin(2x)dx \qquad \text{(c)} \int_{x=-2}^{2} \frac{1}{\sqrt{4-x^2}} dx$$

2. Tell which of these integrals converge and which do not, and give reasons. Of those integrals that converge, calculate the exact values for those that can be written in closed form.

$$\text{(a)} \int_{x=0}^{1} \frac{x+1}{\sqrt{x}} dx \qquad \text{(b)} \int_{x=0}^{1} \frac{\sqrt{x}}{x+1} dx \qquad \text{(c)} \int_{x=\pi}^{\infty} \frac{\sin(\sin x)}{x^2} dx$$

$$\text{(d)} \int_{x=0}^{\infty} xe^{-x} dx \qquad \text{(e)} \int_{x=-\infty}^{0} xe^{-x} dx \qquad \text{(f)} \int_{x=-\infty}^{\infty} xe^{-x} dx$$

3. Compute the total force in pounds of a square plate of glass that forms a vertical wall of a freshwater aquarium. The length of one side of this plate is 6 feet. (Note: The density of water is 62.4 pounds per cubic foot.)

4. Compute the volume of the solid obtained by rotating the portion of the curve $y = e^x$ between $x = 0$ and $x = 1$ about the line $y = -1$.

5. (a) Use the trapezoid rule with 1, 2, and 4 subintervals to compute approximations to the integral in Problem 1(a). Carry 4 digits in the computations.
   (b) Compare your approximations to the value obtained by computing a numerical value for your answer from Problem 1(a).

6. (a) Write down the Taylor polynomial $p_8(x)$ of degree 8 for e^{-x^2}, centered at $x = 0$.
   (b) Write down a bound for the error $e^{-x^2} - p_8(x)$ for $-1 \le x \le 1$.
   (c) Evaluate $\int_{x=-1}^{1} e^{-x^2} dx$ approximately by replacing e^{-x^2} by $p_8(x)$ in the integrand.
   (d) Do you think your approximation is good? Why?