Math. 270-05
Fall, 2005
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Third Examination
Tuesday, October 25, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points. You may keep this exam sheet.

1. For the following functions f, compute df/dx by hand. You will not be given full credit unless you show all steps in your computations.

   (a) $f(x) = \sin(x^2)$ (b) $f(x) = \dfrac{x^2+1}{x^3+1}$ (c) $f(x) = \log(x^2 \sin(x))$

2. Suppose a balloon is rising from sea level, and that the height of the balloon at time t minutes from its launching is approximately $h(t) = 10000\left(1 - e^{-t/60}\right)$ feet. Suppose further that the temperature at height h feet above sea level at the site of the balloon launching is approximately $T(h) = 90 - 0.7\sqrt{h}$ degrees Fahrenheit.

   (a) What is the altitude one hour after launching?

   (b) At what rate, in degrees per minute, is the temperature in the balloon decreasing one hour after launching?

3. If $f(x) = (\sin(x))^3$ for $x \in [-(\pi/2)^3, (\pi/2)^3]$, then compute the instantaneous rate of change of $f^{-1}(y)$ with respect to y when $y = (\sqrt{2}/2)^3$.

4. Consider the function
$$f(x) = \frac{1}{3}x^3 - x.$$
   Use $f'(x)$ to answer the following questions. For full credit, be sure to explain fully.

   (a) Is f increasing or decreasing for $x \in [-1, 1]$?

   (b) If $g(x) = (1/3)x^4 - x$, then $g(1) = f(1) = 0$. Explain why $g(x) \geq f(x)$ for $x \geq 1$.