Math. 270-05
Fall, 2005
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First Examination

Monday, September 12, 2005

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points. You may keep this exam sheet.

1. In 1626, Peter Minuit purchased the island of Manhattan (on behalf of the Dutch West India Company) from the indigenous residents for trinkets worth approximately \$24.00. Suppose that, instead, Mr. Minuit had put his money in the stock market, with an average return of 10% per year,

   (a) Derive a function $A(t)$ that expresses the amount of money A that Mr. Minuit (or his heirs) would have in their account t years after 1626.

   (b) What is $A(1776-1626)$?

   (c) What is $A(2005-1626)$?

   (d) Do you think Peter Minuit would have been better off purchasing Manhattan or putting his money in the stock market? For comparison, the present total worth of goods and services produced in all of the United States per year is approximately $\$11,750,000,000,000 = 11.75 \times 10^{12}$ dollars.

   (e) Do you think this reasoning is appropriate, when applied over such a long period of time? What assumptions that we used when we wrote down $A(t)$ might have been violated?

2. Arrange the following in order of increasing rate of growth, as $x \to \infty$.

   (a) $f(x) = \sqrt{x}$ (b) $f(x) = \log x$

   (c) $f(x) = x^4$ (d) $f(x) = e^{5x}$

   (e) $f(x) = x^2$ (f) $f(x) = x^3 + 1,000,000,000,000x + 1$

   (g) $f(x) = .0000000000001e^x$ (h) $f(x) = 1,000,000,000e^{0.0000000001x}$

   (i) $f(x) = \dfrac{x^2+1}{x^2-2x+1}$

3. Solve the following equation for t.
$$2^{24} = 10^t.$$

4. Provide numbers equal to the following limits, if they exist, but say so if the limit does not exist.

   (a) $\displaystyle\lim_{x\to\infty} \frac{x^2-1}{x^2+1}$ (b) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x^2+1}$ (c) $\displaystyle\lim_{x\to\infty} \sin(x)$

   (d) $\displaystyle\lim_{x\to\pi/2} \tan(x)$ (e) $\displaystyle\lim_{x\to\infty} \frac{\ln(x)}{x}$ (f) $\displaystyle\lim_{x\to\infty} \frac{x}{\ln x}$