Math. 270-05
Fall, 2005
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# Final Examination

*Wednesday, December 7, 2005 10:15AM to 12:45 PM*

**Instructions:** This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 16 points, and 4 points are free. You may keep this exam sheet.

1. Radioactive carbon 14, produced by interaction of carbon dioxide in the air with cosmic rays, is absorbed by all living things, and the proportion of carbon 14 to regular carbon 12 in plants and animals is almost constant while the plant or animal is alive. However, after the plant or animal dies, the carbon 14 in its tissue decays exponentially, with a half-life of 5700 years.
   (a) Write down an equation for the percentage of carbon 14 left in the tissue after $t$ years.
   (b) At an archaeological site in New Mexico, charcoal from an ancient fire is determined to have 29% of the original carbon 14 remaining. Approximately how long ago did the fire burn?

2. Find a value of the constant $k$ such that
$$\lim_{x\to 1} \frac{x^2 - kx + 4}{x - 1}$$
exists.

3. An object is oscillating at the end of a spring. Its position, in centimeters, relative to a fixed point, is given as a function of time $t$ in seconds by
$$y = y_0 \cos(2\pi\omega t), \quad \text{with } \omega \text{ a constant.}$$
   (a) Find an expression for the velocity and acceleration of the object.
   (b) Show that the function $y$ satisfies the differential equation
$$\frac{d^2y}{dt^2} + 4\pi^2\omega^2 y = 0.$$

4. A spherical cell is growing at a constant rate of 400 cubic micrometers per day. At what rate is its radius increasing when the radius is 10 micrometers?

5. Find parametric equations for a circle of radius 3 centered at the point $(1, 2)$ and traced counter-clockwise.

6. (a) Graph $f(x) = x(x+1)(x-2)$.
   (b) Find the total area between the graph and the $x$-axis between $x = -1$ and $x = 2$.
   (c) Find $\int_{-2}^{1} f(x)dx$ and interpret it in terms of areas.