Final Examination

Thursday, May 15, 2003, 7:30AM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. You may keep this exam sheet. Each problem is worth 12 points, and 4 points are free.

1. Write down a vector of length 1 pointing in the direction in which \( f \) is increasing most rapidly at the point \((1, 2)\), where \( f(x, y) = x^2 - y^2 \).

2. Write down a parametrization of the circle of radius 1 centered at the point \((1, 1)\).

3. Find and classify all critical points of the function
\[
f(x, y) = \frac{x^3}{3} - xy^2 - y^2.
\]

4. Evaluate the following integral.
\[
\int_{x=-4}^{4} \int_{y=-\sqrt{16-z^2}}^{\sqrt{16-z^2-y^2}} \int_{z=-4}^{\sqrt{16-z^2-y^2}} z \, dx \, dy \, dz
\]

5. Write down parametric equations for the plane through the points \((1, 1, 1)\), \((1, 0, 0)\), and \((0, 1, 0)\).

6. Write down a single equation for the plane in Problem 5. Show all work. Also, check that the two representations represent the same plane.

7. Compute the work required to move a particle along the portion of the spiral \( x = \cos(t) \), \( y = \sin(t) \), \( z = t \) for \( 0 \leq t \leq \pi \), through the vector field given by \( \vec{F}(x, y, z) = (x, y, z)^T \).

8. Use Green’s theorem to compute
\[
\oint_{C} \vec{F} \cdot d\vec{r},
\]
where \( \vec{F}(x, y) = (y, -x)^T \) and where \( C \) is the circle of radius 1 centered at \( (x, y) = (0, 0) \).