Fourth Examination
Friday, April 20, 2001

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. That is, show all work. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points.

1. Compute
\[ \int \int_{\mathcal{R}} xy \, dA, \]
where \( \mathcal{R} \) is the region bounded by the \( x \)-axis, the curve \( y = x^2 \), and the line \( x = 1 \).

2. Compute the following integral, changing the coordinate system if necessary.
\[ \int_{x=-\sqrt{\pi}}^{\sqrt{\pi}} \int_{y=-\sqrt{\frac{\pi-x^2}{\pi-x^2}}}^{\sqrt{\pi-x^2}} \sin(x^2 + y^2) \, dy \, dx \]

3. Compute the mass of a ball of radius 4000 whose density \( \rho \) units from the center is \( 1/\rho^2 = 1/(x^2 + y^2 + z^2) \). (Hint: use the appropriate coordinate system to set up the iterated integral.)

4. Find the average value of the sum of squares of three numbers, where each number is between 0 and 2.