Final Examination
Wednesday, May 2, 2001, 1:30 PM–4:00PM

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. That is, show all work. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each part of each problem is worth 20 points.

1. Which of the following functions does the surface in Figure 1 represent? Explain fully why you chose the function you did.

   a) \( f(x, y) = x^2 + y^2 \)  
   b) \( f(x, y) = x^2 \sin(y) \)  
   c) \( f(x, y) = x + y \)  
   d) \( f(x, y) = e^x - e^y \)  
   e) \( f(x, y) = x^2 + \sin(y) \)  
   f) \( f(x, y) = x^2 \cos(y) \)

2. Consider the plane through the points \((1,0,0), (1,1,0), \) and \((1,1,1)\).
   (a) Write down parametric equations for this plane.
   (b) Also write down a single equation for the plane (relating \( x, y, \) and \( z \)).

3. The temperature at a point \((x, y, z)\) in space is given by

\[
T(x, y, z) = 70 - 0.001x + 0.005y - 0.5z^2.
\]
Also, the position of an object at time $t$ is given by

$$\vec{r}(t) = (100t^2, -100t^2, 2.5\sqrt{t})$$,

for $1 \leq t \leq 5$.

(a) Compute the temperature $T$ observed by the object at time $t = 4$.

(b) Use the chain rule for functions of several variables to compute the rate of change of temperature observed by the object at time $t = 4$.

(c) Write down an integral that represents the distance the object has travelled between $t = 0$ and $t = 4$.

4. Consider the function $f(x, y) = x^2 - y^2 + xy$.

(a) Write down the tangent plane approximation to $f$ at the point $(x, y) = (1, 2)$.

(b) Use the tangent plane approximation or differential to get an approximation to the range of values of $f$ as $x$ ranges between 0.9 and 1.1 and $y$ ranges between 1.9 and 2.2. (Hint: Consider $\Delta x = -0.1$, $\Delta x = +0.1$, $\Delta y = -0.1$, and $\Delta y = +0.1$.)

5. Compute the average value of $f(x, y, z) = e^{-(x^2+y^2+z^2)^{3/2}}$ over the ball of radius 2 centered at the origin.

6. Sketch the vector field $\vec{F}(x, y) = (-y, x)$ within the rectangle $-2 \leq x \leq 2$, $-2 \leq y \leq 2$. 