Fourth Examination
Thursday, November 13, 2003

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points. You may keep this exam sheet.

1. Use polar coordinates to find

\[ \int \int_C y \, dA, \]

where \( C \) is the portion of the disk \( x^2 + y^2 \leq 1 \) with \( y \geq 0 \).

2. Compute the integral from Problem 1, but use rectangular coordinates instead of polar coordinates.

3. Compute the length of the portion of the helix

\[
\begin{align*}
x(t) &= \frac{1}{\sqrt{2}}\{t \sin(t) + \cos(t)\}, \\
y(t) &= \frac{1}{\sqrt{2}}\{-t \cos(t) + \sin(t)\}, \\
z(t) &= \frac{1}{2\sqrt{2}}t^2,
\end{align*}
\]

for \( 0 \leq t \leq 1 \).

4. Consider two objects, one whose position at time \( t \) is \( x(t) = t, y(t) = 2t, z(t) = 4t \), and the other whose position at time \( t \) is \( x(t) = t - 1, y(t) = t^2 - 2t + 1, z(t) = (t - 1)^3 \).
   
   (a) Do the two objects collide? If so, then at what time \( t \) and where?
   
   (b) Do the objects’ paths cross? If so, then where?