Fourth Examination
Friday, November 9, 2001

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points.

1. Find and classify all critical points of \( f(x, y) = x^2 - 2xy + 2y^2 \).

2. State whether or not each of the following functions has a global maximum, a global minimum, or both. In all cases, state why.

   (a) \( f(x, y) = \frac{x^2y^2}{1 + x^4y^4} \)

   (b) \( f(x, y) = xy \)

   (c) \( f(x, y, z) = -x^2 - y^2 - z^2 \)

3. Find all global maxima and minima of \( f(x, y) = x + y^2 \) over the unit disk \( x^2 + y^2 \leq 1 \).