Third Examination  
Tuesday, October 9, 2001

Instructions: This exam should be done on your own paper. Your name should be on each sheet and on the back of the last sheet; the answers should appear written carefully and in order. If in doubt, show intermediate steps: Full credit may not be given, even for correct answers, unless work is arranged clearly and explained. This exam is closed book. You may leave after handing in your exam paper, but be sure to check your answers carefully. Each entire problem is worth 25 points.

1. For 
\[ f(x, y) = e^{xy} \cos(x), \]
compute 
(a) \[ \frac{\partial f}{\partial x}, \]  
(b) \[ \frac{\partial f}{\partial y}, \]  
(c) \[ \frac{\partial^2 f}{\partial x^2}, \]  
(d) \[ \frac{\partial^2 f}{\partial y^2}, \]  
and (e) \[ \frac{\partial^2 f}{\partial x \partial y}. \]

2. Suppose you are hiking on a mountain. You have an altimeter, a compass, and a balance to measure angles. Suppose, at a point on the mountain, you measure your altitude to be 7135 ft. Also suppose that, using the compass and balance, you measure your rate of increase of altitude to be 1200 feet per 5000 feet travelled as you go east and 2500 feet per 5000 feet travelled as you go north. What will be your approximate change in altitude if you head southeast for 100 feet?

3. Let the function in Problem 2 be called \( A(x, y) \), where \( A \) is the altitude \( x \) feet east and \( y \) feet north of your present position.
   (a) Write down the gradient \( \nabla A(0, 0) \).
   (b) What is the directional derivative \( D_{\vec{u}} A(0, 0) \) of \( A \) at the point \( (0, 0) \) in the direction \( \vec{u} = (1/\sqrt{2}, -1/\sqrt{2}) \)?

4. The temperature in degrees Fahrenheit \( x \) miles east, \( y \) miles north, and \( z \) miles above a certain airport runway is modelled by 
\[ T(x, y, z) \approx 85 + 0.01x - 0.01y - 10z^2. \]
An airplane takes off from the runway, with a trajectory given for the first hour of flight by 
\[ x(t) = 100t + 300t^2, \]
\[ y(t) = 50t + 150t^2, \]
\[ z(t) = 8t - 2t^2, \]
where \( t \) is the time in hours.
   (a) What is the position of the plane in relation to the runway after half an hour?
   (b) What is the temperature outside the plane at that time?
   (c) At what rate is the temperature outside the plane increasing or decreasing half an hour into the flight? Give both the numerical value and answer whether it represents an increase or decrease.