Linear Interval Equations: The Role of Preconditioning

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It is well known that the exact interval hull $[x, \overline{x}]$ of the solution set of a system of linear interval equations

$$Ax = b$$

(1)

is NP-hard to compute [1]. On the other hand, the interval hull $[x, \overline{x}]$ for the preconditioned system

$$A^{-1}_c Ax = A^{-1}_c b$$

(where $A_c$ is the midpoint of $A$) can be computed in polynomial time with only two matrices to be inverted [2, 3]. Since $[x, \overline{x}] \subseteq [x, \overline{x}]$, we obtain an enclosure $[x, \overline{x}]$ of the solution set of (1) whenever the procedure is applicable (which is known to be the case if and only if $A$ is strongly regular).

Nevertheless, the main question has remained unanswered so far: how well does $[x, \overline{x}]$ approximate the exact hull $[x, \overline{x}]$ of (1)? In this talk we shall present explicit formulae for nonnegative vectors $\bar{d}$ and $\overline{\bar{d}}$ computable in polynomial time such that

$$x \leq \underline{x} \leq \underline{x} + \bar{d},$$

$$\overline{x} - \overline{\bar{d}} \leq \overline{x} \leq \overline{x}$$

hold. The formulae for $\bar{d}$ and $\overline{\bar{d}}$, obtained in a rather sophisticated way, aside from their computational value also allow to draw several theoretical consequences.

References
