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UNICALC AS A TOOL FOR SOLVING PROBLEMS WITH INACCURATE AND SUB-DEFINITE DATA

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We consider the problem solver UniCalc which is based on a new mathematical approach. UniCalc is designed for solving any system of algebraic and algebraic-differential equations and inequalities, with real and/or integer interval parameters . The architecture of the solver is described and results of numerical experiments and system demands are given.

UNICALC КАК СРЕДСТВО РЕШЕНИЯ ЗАДАЧ С НЕТОЧНЫМИ И ЧАСТИЧНО ОПРЕДЕЛЕННЫМИ ДАННЫМИ

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Рассматривается решатель задач UniCalc, основанный на новом математическом подходе. UniCalc разработан для решения произвольных систем алгебраических и алгебро-дифференциальных уравнений и неравенств, параметры которых могут быть действительными и/или целыми интервалами. Описана архитектура решателя. Приводятся результаты численных экспериментов и требования системы.

1. Purpose and capabilities

UniCalc is intended to solve arbitrary systems of algebraic and algebraic-differential relations. A relation is understood to be an equation, inequality or a logic expression. The system to be solved may be over- or underdetermined, and the system parameters: coefficients, unknowns, constants. initial conditions in the Cauchy problem may be specified inaccurately (as intervals). Both real and integer interval parameters are acceptable. There is no need to specify any initial approximation. The result is either a parallelepiped guaranteed to contain all roots of the system, or a message that the system is unsolvable. If the system has a unique solution, then the parallelepiped is reduced to a single point (with a specified accuracy).

2. Solver architecture

The solver is an integrated environment allowing system solution, modification, calculation, result viewing, accuracy setting, etc. To input and modify the system, the solver contains a Turbo-compatible editor. The user interface is organized as a multiwindow shell enabling the simultaneous viewing of several models and a comparison of the results obtained. The solver has an input language making it possible to represent the system in a form that is close to normal mathematical formalism. All problems (both direct and inverse in the real and integer number fields, both precise and interval ones) are solved by a single algorithm. The solution procedure is implemented by a data-flow calculator that interprets a network of a special kind. To translate the input language into the network, the solver includes a translator and a number of preprocessors including a symbolic transformation preprocessor and a preprocessor for solving algebraic-differential relations. The first preprocessor simplifies the system, performs symbolic differentiation and operations required to solve linear systems; the second preprocessor translates the system from the language used for writing differential equations to the basic UniCalc language, supports the procedure for solving systems of this type and ensures graphical display of solutions.

3. Calculator

The UniCalc calculator [2,3] implements the concept of the Generalized Calculation Model (GCM) [1] which is related to data-flow knowledge processing models. A GCM is a functional network with a prespecified scheduling of operation execution. The functional network is a bipartite graph having vertices of two types: objects and operators. The objects represent the system's parameters, operators specify functional links, and each operator is associated with a procedure to interpret it. The order of executing these procedures is determined by the scheduler, and is, in general, a parallel, asynchronous, nondeterministic process. UniCalc runs these procedures in a defined order and, to process sub-definite values that are objects of the network, the algorithms of interval arithmetics are used.

4. Solving systems of algebraic-differential equation

One of the most amazing of UniCalc's features is its capability of solving systems of algebraic-differential equations with imprecise parameters [4]. An algebraic-differential system is understood as a system of relations, including the Cauchy problem for the systems of the first-order ordinary differential equations, algebraic equations, inequalities and logical expressions. Relation parameters and Cauchy problem boundary conditions may be imprecise. Such systems need processing in an individual window. To write these systems, the basic UniCalc language is supplemented by special tools. To approximate derivatives, the preprocessor provides various explicit and implicit schemes. The user has an opportunity to choose the scheme, type and value of the integration step, points for displaying the results, the accuracy, etc. The results are issued as an interval array for each variable. The solution process is accompanied by displaying a band determined by the upper and lower boundaries of its value intervals.

5. Numerical experiments

To evaluate the efficiency of the solver operation and to determine the range of its application, a large number of problems have been tested. These include: linear and nonlinear systems of equations, inequalities, mixed systems, various kinds of integer problems, optimization problems, interval problems, systems of differential equations, etc. In particular, almost all the test considered in [5,6] have been solved. The UniCalc consistently yielded narrower intervals than those given in the tests as an initial approximation. Automatic root search methods have been used to find individual roots. A problem proposed as a test in paper [7] has been solved successfully in the initial problem statement without its reduction to another system. UniCalc has shown very good efficiency in solving various integer problems: solving Diophantine problems, integer optimization problems, etc. Optimization problems for known functions including the Rozenbrock (functions up to the 15th order) and the Powell function have been under consideration. However, such testing has shown UniCalc weaknesses: UniCalc is inefficient in solving linear systems of equations. To eliminate this drawback, the analytical preprocessor is now being used. For testing the preprocessor on systems of algebraic-differential relations, various problems have been under consideration, including well- and weakly-defined, strict, oscillating problems, etc. These tests have proven successful even for problems with sub-definite parameters. However, certain problems intrinsic to interval methods remain unsolved.

6. Hardware requirements

UniCalc runs on IBM-compatible computers under the MS-DOS operating system, version 3.0 and higher. The necessary RAM size is 450 **KB**. The number of relations in the system to be solved may range up to 300 relations. The elapsed time is from several milliseconds to several minutes.

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