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Edited by **R. VICHNEVETSKY**
Rutgers University
New Brunswick, USA

P. BORNE
IDN
Villeneuve d'Ascq, France

J. VIGNES
Université Pierre et Marie Curie
Paris, France

THE INVERSE PROBLEM OF ELECTROENCEPHALOGRAPHY ASSUMING DOUBLE LAYER NEURAL GENERATORS

Robert D. Sidman, Ph.D., R. Baker Kearfott, Ph.D.
Department of Mathematics, University of Southwestern Louisiana
Lafayette, Louisiana 70504 USA

Christine Schlichting, Ph. D.
Vision Laboratory, NSMRL, Naval Submarine Base
Groton, Connecticut 06349 USA

Abstract

The equivalent dipole source for scalp-recorded evoked potential components closely resembles the actual neural generators of these components when these generators consist of a localized population of neurons firing synchronously. For example, the N30-P30 response to median nerve stimulation is thought to be generated in the posterior bank of the central fissure and the location and orientation of the equivalent dipole source for this component agree rather well with this presumed cortical generator (1). Sources of other EP components, however, are more widely distributed and are better simulated by extended dipole layers. Such examples include the primary response to flash or pattern reversal photic stimulation, the N1 and P2 responses to auditory stimulation, and possibly the P300 auditory response.

The purpose of this paper is to describe the application of the inverse problem to some of the data mentioned above, assuming that measured scalp data are generated by a double layer on the surface of the brain rather than by a single equivalent current dipole. This approach has the advantage of leading to well-conditioned numerical calculations in cases where the assumption of a single dipole source yields an ill-conditioned problem.

DLM

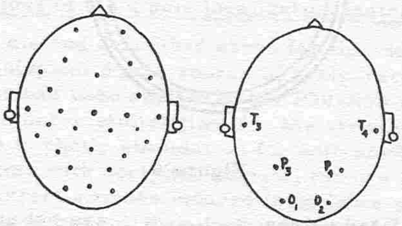
The dipole localization method (DLM) has been described in detail elsewhere (2). Briefly, suppose that the quantities $V_k(t)$, $k = 1, \ldots, n$, are evoked potentials measured at electrode sites A_k, t milliseconds after a stimulus. Also, let $V_k(D)$ be the theoretical voltage that would be produced at A_k by the current dipole D, where D can be described by six parameters--three giving its location and three giving its moments. DLM consists in numerically constructing the dipole $D(t)$ which minimizes in a least-squares sense the difference between theoretical and measured voltages at each latency t. That is, one minimizes for each t, the quantity $\mathrm{RHO}(D(t))$, where

$$\mathrm{RHO}(D(t)) = \sum_{k=1}^{n} (V_k(t) - V_k(D(t)))^2 / \sum_{k=1}^{n} V_k^2(t).$$

If RHO is uniformly "small" during a time epoch for which the scalp recorded potentials are generated by a localized synchronous neural source, then the time varying dipole, which minimizes RHO and represents the superposition of many neural units, will usually have a stable location and direction. In early studies (1), the empirical condition, $\mathrm{RHO} < 0.1$ during an epoch generally implied a stable equivalent source.

This condition depends upon the number of leads that are used and seems to be a reasonable sufficient condition for equivalent source stability when that number is sufficiently large and the majority of leads do not lie over the 0-potential curves of the scalp topography.

We illustrate these considerations by two applications of DLM. This method is applied to the potentials evoked by pattern-reversing checkerboard stimulation. Figures 1a and 1b show, respectively, the twenty-eight and six scalp electrode sites used in two studies, with a linked-ears reference.



(a) (b)

Figure 1

The following pair of graphs (Figure 2) are the "power" curves for these sets of data. The maxima occur at those latencies when there is synchronous neural activity. Equivalent dipole sources are then constructed for time intervals containing those latencies. For the purposes of this paper only, the starred (*) power peaks are analyzed.

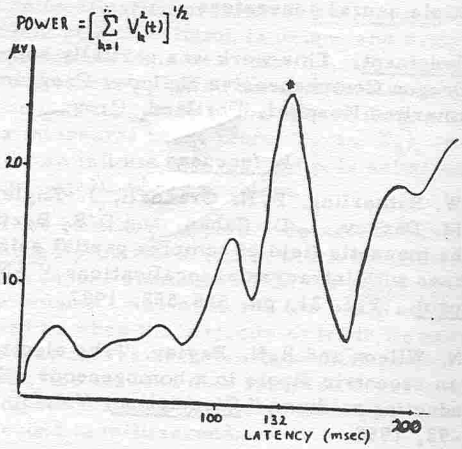


Figure 2(a)

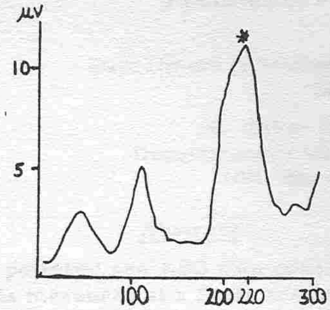


Figure 2(b)

Figure 3a shows the dipole source for the 28-lead data during the time interval 132-144 msec. More precisely, it is the average dipole for this epoch. RHO is less than 0.1 and the dipole locations vary very little (within a volume of radius 2 millimeters) throughout this time interval.

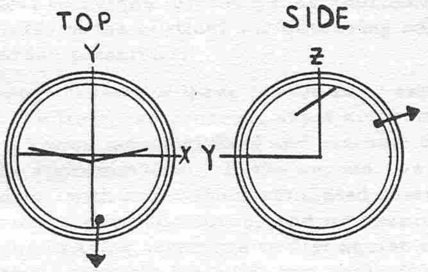


Figure 3(a)

Figure 3b is quite different. In this case, $RHO < 0.01$ during a 5 msec epoch containing the latency 220 msec, but the dipole location is very unstable. In addition, these locations are frontal--inconsistent with the assumption that the potentials are generated in the visual cortex.

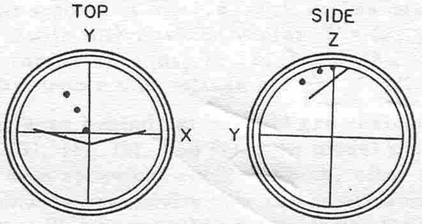


Figure 3(b)

The fact that RHO is less than 0.01 should not be surprising since there are six dipole parameters to be calculated and precisely six scalp electrode sites. The equations, $V_k(t) = V_k(D(t))$ could be solved exactly for $k = 1, \ldots, 6$ to give $RHO(D(t)) = 0$ for every t in this time interval. This is apparently an ill-conditioned problem since small variations in $V_k(t)$ lead to large variations in $D(t)$.

Dipole Layers

Instead of simulating the neural generators by a single current dipole, we now model them by a dipole layer on the boundary of a sphere. The particular form of this layer is illustrated in Figure 4. It is characterized by five parameters: the radius r of the sphere on which the layer lies; a quantity, "intensity," which is a measure of the dipole density of the surface element; spherical angles θ and ϕ giving the direction of the central axis of the surface element; and the angle W giving the angular extent of the surface element. The angles θ and ϕ are related to Cartesian coordinates in the usual way, where the positive x-axis passes through the right ear, the positive y-axis passes through the nasion, and the positive z-axis passes through the vertex. If the layer is assumed to lie on the surface of the brain, then r is fixed and the equivalent layer is determined by four parameters. The inverse problem, assuming such a source, is outlined in (3).

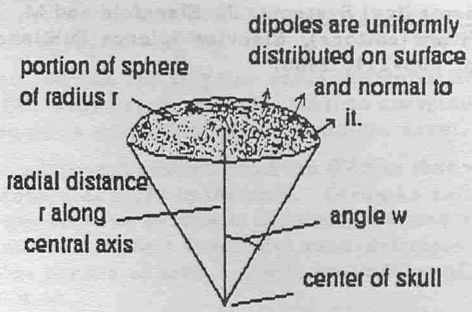


Figure 4

If this method is applied to the three time points at which DLM was used for the six-lead data (Figure 3b), it yields the following stable layers:

Time Point	θ	ϕ	Intensity	W
1.	-25.6°	15.6°	.072	95.8°
2.	-25.1°	15.1°	.073	96.2°
3.	-24.4°	14.2°	.073	96.5°

This result is illustrated in the following figure.

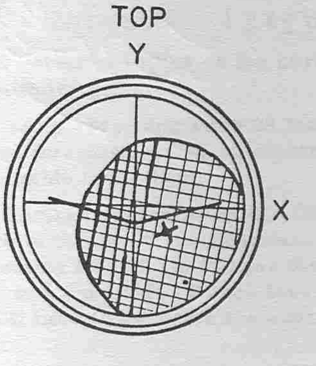


Figure 5

We shall discuss the application of this model to other cases as well and relate the results to the neurophysiological bases of several evoked potential components.

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References

(1) R. D. Sidman, "The time-dependent equivalent dipole source for the response to median nerve stimulation," IEEE Trans. Biomed. Eng. Vol. 31, No. 6, pp. 481-483, 1984.

(2) R. D. Sidman, "The inverse problem of electroencephalography--mathematical overview and current research," Proceedings of 12th IMACS Congress.

(3) R. D. Sidman and D. B. Smith, "Localization of the neural generators of scalp-recorded potentials by means of mathematical models," Modelling of Biomedical Systems, J. Eisenfeld and M. Witten (editors), Elsevier Science Publishers, pp. 205-211, 1986.