Department of Mathematics
The University of Southwestern Louisiana
Ph.D. Preliminary Examination: Numerical Analysis
Wednesday, August 25, 1993 1:00 – 5:00 PM, MDD 206

Instructions: This portion of the exam is open-book, open notes. Make sure to organize and check your work. Three hours should be plenty of time for this exam, although you should be able to finish well before then.

1. Compare iterative methods and direct methods for solving linear systems of equations. Which class of method would be preferable if the matrix had a sparse but complicated structure, and you were using a single instruction multiple data machine, such as a Connection machine?

2. Use an interval technique to prove that the system of equations

$$f_1(x) = x_1^2 - 2e^{x_2} = 0$$
$$f_2(x) = x_2^2 - 2e^{x_1} = 0$$

has no solutions within the region bounded by $0 \le x_1 \le 1$ and $0 \le x_2 \le 1$.

3. Use the generalized method of automatic differentiation to compute the gradient of

$$f(x,y) = x^2 \sin(y) + y^2$$

at $(x,y) = \left(1, \frac{\pi}{4}\right)$. Write down the matrix and the intermediate steps, as if you were giving the example to a class.

4. Discretize

$$u_{xx} = -u \qquad u(0) = u(1) = 0$$

with a finite element method with basis functions

$$\phi_1(x) = \begin{cases} \cos\left(\frac{3}{2}\pi(x - \frac{1}{3})\right) & \text{if } 0 \le x \le \frac{2}{3} \\ 0 & \text{otherwise,} \end{cases}$$
$$\phi_2(x) = \begin{cases} \cos\left(\frac{3}{2}\pi(x - \frac{2}{3})\right) & \text{if } \frac{1}{3} \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Contrast the system you get with the one you get when you discretize using two piecewise linear functions, centered at $x = \frac{1}{3}$ and $x = \frac{2}{3}$.

5. Describe two methods solving the least squares problem:

$$Az = b$$

where $A \in \mathbb{C}^{m\times n}$, $z \in \mathbb{C}^{n\times 1}$, $b \in \mathbb{C}^{m\times 1}$, and apply one of them to find the least squares solution of

$$\begin{bmatrix} 3 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

6. Set up an implicit finite difference scheme with three internal mesh points in space for the following parabolic initial-boundary value problem

$$\begin{cases} u_t = u_{xx} + 2u, \ 0 < x < 1, \ 0 < t < T, \\ u(0,t) = f(t), \ u(1,t) = g(t), \ 0 < t < T, \\ u(x,0) = h(t), \quad 0 \le x \le 1, \end{cases}$$

and carry out a stability analysis.

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