1. We first stretch the x_2 axis by 3 with the transformation whose matrix is $T_1 = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$.

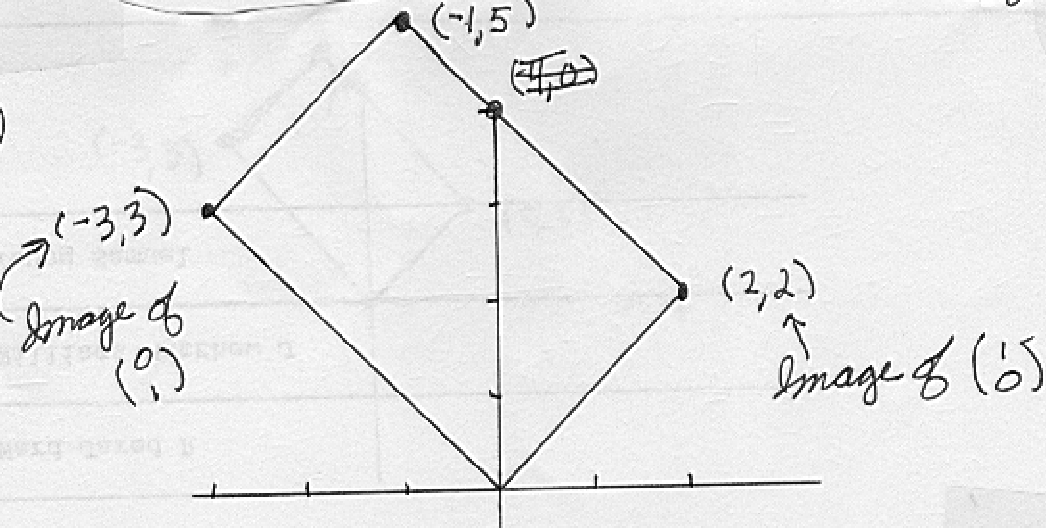
We then rotate clockwise by $\pi/6$ (that is, through an angle of $-\pi/6$) ~~to get the~~ with the transformation

$$T_2 = \begin{pmatrix} \cos(-\pi/6) & -\sin(-\pi/6) \\ \sin(-\pi/6) & \cos(-\pi/6) \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix}.$$

The matrix for the transformation is thus:

$$T_2 T_1 = \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 3/2 \\ -1/2 & 3\sqrt{3}/2 \end{pmatrix}$$

2.



3 We compute A^{-1}:

9

$$\left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|cc} 1 & -3/2 & 1/2 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 6 & -1 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{cc|cc} 1 & -3/2 & 1/2 & 0 \\ 0 & 1 & -1/6 & 1/6 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & 1/4 & +1/4 \\ 0 & 1 & -1/6 & 1/6 \end{array}\right)$$

Thus, $A^{-1} = \begin{pmatrix} 1/4 & 1/4 \\ -1/6 & 1/6 \end{pmatrix}$

---

4 b $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ -1/6 & 1/6 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

c $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 \\ -1/6 & 1/6 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

---

4 9 We compute the LU factorization:

We may use ~~the~~ computations ~~from~~ similar to 39.

In particular, the matrix L is computed as follows:

$$\begin{pmatrix} 2 & -3 \\ 2 & 3 \end{pmatrix} \underset{\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}}{\xrightarrow{R_1 \leftarrow R_1/2}} \begin{pmatrix} 1 & -3/2 \\ 2 & 3 \end{pmatrix} \underset{\begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}}{\xrightarrow{R_2 \leftarrow R_2 - 2R_1}} \begin{pmatrix} 1 & -3/2 \\ 0 & 6 \end{pmatrix}$$

Thus, $A = LU$, where $L = \begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}$ and $U = \begin{pmatrix} 1 & -3/2 \\ 0 & 6 \end{pmatrix}$.

4b Since $A = LU$, to solve $LUx = b$, we first solve $Lw = b$ with "forward substitution", then solve $Ux = w$ with "back substitution":

$$\begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \Rightarrow \quad w_1 = 2/2 = 1, \quad w_2 = \frac{2 - 2w_1}{1} = 0.$$

$$\begin{pmatrix} 1 & -3/2 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \quad 6x_2 = 0 \Leftrightarrow x_2 = 0;$$

$$x_1 = \frac{1 - 3/2\,x_2}{1} = 1.$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

4C We proceed as in **4b**:

$$\begin{pmatrix} 2 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \end{pmatrix} \Rightarrow w_1 = -3/2 \Rightarrow w_2 = \cancel{3}.\ \frac{3+3}{\cancel{1}} = \cancel{3}6$$

$$\begin{pmatrix} 1 & -3/2 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -3/2 \\ \cancel{3}6 \end{pmatrix} \Rightarrow x_2 = 1 \Rightarrow x_1 = 0$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
