

Third Exam

Thursday, July 14, 2005

This exam is closed book. Make sure your name is on all pages. You should put your work on your own paper, and you may keep this exam sheet upon leaving. Be sure to check your work carefully, and to show intermediate computations in a logical presentation. **Full credit will not be given unless computations are shown.** Each entire problem is worth 25 points.

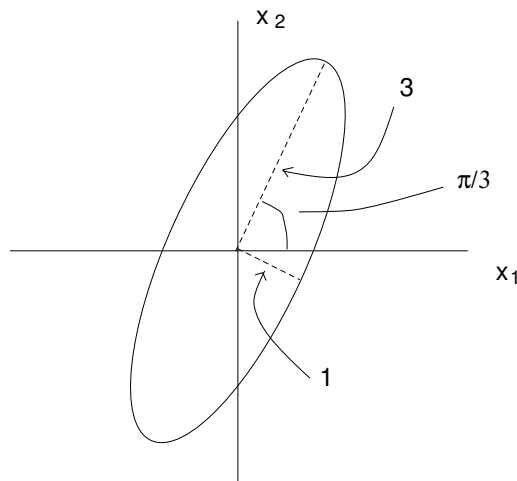


Figure 1: The image for problem 1

1. Write down the matrix for a linear transformation that maps the circle $x^2 + y^2 = 1$ onto the ellipse in Figure 1. Show all of your work.
2. Draw the image of the unit square $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$ under the transformation $x \leftarrow Ax$, where

$$A = \begin{pmatrix} 2 & -3 \\ 2 & 3 \end{pmatrix}.$$

Label the sizes of the parts of the image and/or the coordinates of the vertices of the image.

3. For the matrix A as in Problem 2, do the following:
 - (a) Compute A^{-1} .
 - (b) Use A^{-1} to find the solution x to $Ax = (2, 2)^t$.
 - (c) Use A^{-1} to find the solution x to $Ax = (-3, 3)^t$.
4. For the matrix A from Problem 2, do the following:
 - (a) Compute the LU factorization of A .
 - (b) Use the LU factorization to find the solution x to $Ax = (2, 2)^t$.
 - (c) Use LU factorization to find the solution x to $Ax = (-3, 3)^t$.