1. $\theta = \arccos\left(\dfrac{\vec{u}\circ\vec{v}}{\|\vec{u}\|\|\vec{v}\|}\right)$. $\vec{u}\circ\vec{v} = (1,-1,1)\circ(1,2,3) = 1-2+3 = \boxed{2}$

$\|\vec{u}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$, $\|\vec{v}\| = \sqrt{1+4+9} = \sqrt{14}$.

$\theta = \arccos\left(\dfrac{2}{\sqrt{3}\sqrt{14}}\right) = \arccos\left(\dfrac{2}{\sqrt{42}}\right) \approx \cancel{.9056}\ 1.257$ radians

(about $\cancel{51.9}\ 72^\circ$)

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2. The component of $\vec{u}$ in the direction of $\vec{a}$ is:

$\dfrac{\vec{u}\circ\vec{a}}{\|\vec{a}\|^2}\vec{a} = \dfrac{\vec{u}\circ\vec{a}}{\vec{a}\circ\vec{a}}\vec{a} = \dfrac{(1,2,3,4,5,6)\circ(1,-1,1,-1,1,-1)}{(1,-1,1,-1,1,-1)\circ(1,-1,1,-1,1,-1)}\vec{a} = \dfrac{1-2+3-4+5-6}{6}\vec{a}$

$= \dfrac{-3}{6}\vec{a} = -\dfrac{1}{2}\vec{a} = \boxed{\left(\dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{1}{2}, \dfrac{-1}{2}, \dfrac{1}{2}\right)}$.

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3. Vectors in the direction of plane are $(2,1,3)-(1,1,1) = (1,0,2) = \vec{v}$

and $(-1,1,-1)-(1,1,1) = (-2,0,-2) = \vec{w}$, so

$\vec{v}\times\vec{w}$ is normal to the plane.

$$\vec{v}\times\vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 2 \\ -2 & 0 & -2 \end{vmatrix} = -\vec{j}\begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix} = -\vec{j}(-2+4) = -2\vec{j}$$

$= (0,-2,0)$.

Thus, an equation for the plane is:

$((x,y,z)-(1,1,1))\circ(0,-2,0) = 0$, i.e. $-2(y-1) = 0$,

or $\boxed{y = 1}$

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4. We'll work with the augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & -1 \\ 4 & 5 & 6 & 1 \end{array}\right] \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & -3 & -6 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 2 & -5/3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 7/3 \\ 0 & 1 & 2 & -5/3 \end{bmatrix}, \text{ giving } \begin{array}{l} x_1 = 7/3 + x_3 \\ x_2 = -5/3 - 2x_3 \end{array}$$

a particular solution to $Ax = b$ is $x = \begin{bmatrix} 7/3 \\ -5/3 \\ 0 \end{bmatrix}$, and a general solution to the homogeneous system is $t\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

5 A vector in the direction of the line is:

$(4,4,4)-(-2,1,3)=(6,3,1)$, so a parametrization of the line segment is:

$$\boxed{\vec{r}(t)=(-2,1,3)+t\,(6,3,1), \quad 0\le t\le 1}$$

6 $$\begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ -1 & 1 & -1 & 1 & 2 \\ 0 & 3 & 2 & 5 & 3 \\ 2 & 1 & 4 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 0 & 3 & 2 & 5 & 3 \\ 0 & 3 & 2 & 5 & 3 \\ 0 & -3 & -2 & -5 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 0 & 1 & 2/3 & 5/3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 5/3 & 2/3 & -1 \\ 0 & 1 & 2/3 & 5/3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus
$$\begin{aligned} x_1 &= -1-\tfrac{5}{3}x_3-\tfrac{2}{3}x_4 \\ x_2 &= 1-\tfrac{2}{3}x_3-\tfrac{5}{3}x_4 \\ x_3 &= x_3 \\ x_4 &= x_4. \end{aligned}$$

Thus $x_p=\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $x_h=t\begin{bmatrix} -5/3 \\ -2/3 \\ 1 \\ 0 \end{bmatrix}+u\begin{bmatrix} -2/3 \\ -5/3 \\ 0 \\ 1 \end{bmatrix}$