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① $\left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \overset{R_1 \leftarrow (-1)R_1}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 2 & 0 & 0 & 1 \end{array}\right]$ for problem 5

$R_2 \leftarrow R_2 - R_1$, $R_3 \leftarrow R_3 - 2R_1$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 2 & 0 & 1 \end{array}\right]$

$R_3 \leftarrow R_3 - 2R_2$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & -2 & 1 \end{array}\right]$

for problem 5

$R_3 \leftarrow \left(\frac{1}{2}\right)R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1/2 \end{array}\right]$

$R_2 \leftarrow R_2 - R_3$, $R_1 \leftarrow R_1 + R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & 1/2 \\ 0 & 1 & 0 & 1 & 2 & -1/2 \\ 0 & 0 & 1 & 0 & -1 & 1/2 \end{array}\right]$

Thus, $\boxed{A^{-1} = \begin{bmatrix} -1 & -1 & 1/2 \\ 1 & 2 & -1/2 \\ 0 & -1 & 1/2 \end{bmatrix}}$.

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② We must have $x+y = x-y \iff y = -y$.

$x = x$

$y = -y$.

Thus, the only condition is $\boxed{y=0}$; x can be anything.

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③ $\begin{vmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 2 \end{vmatrix} = 2(0)(0) + (1)(2)(1) + (-1)(1)(2)$

$- (-1)(2)(0) - (0)(1)(2) - (1)(1)(2)$

$= 0 + 2 - 2 - 0 - 0 - 2 = \boxed{-2}$

4. It is easiest to expand along the second column. We have:

$$\begin{vmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & 3 & 1 \\ -1 & 0 & 1 & 2 \end{vmatrix} = +1 \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - (-1) \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$= 1(6-1) + 1(4+1) = 5+5 = \boxed{10}$$

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5. We may examine the multiplying factors from problem 1. Alternatively, we can proceed as follows:

$$\begin{vmatrix} -1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 2 \end{vmatrix} = -1 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 2 \end{vmatrix} \overset{(R_3 \leftarrow R_3 - 2R_2)}{=} -1 \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$$

$$= (-1)(2) \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (-1)(2) \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$(R_1 \leftarrow R_1 - R_3)$

$$= (-1)(2)(1)(1)(1) = \boxed{-2}.$$

(There are many other correct ways.)
