Math. 362-02, Fall, 2016, First exam answers, page 1.

① (a) The augmented matrix is: $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 6 \\ 1 & 2 & 3 & 3 \end{bmatrix}$

(b) $\underset{R_3 \leftarrow R_3 - R_1}{\overset{R_2 \leftarrow R_2 - 2R_1}{\sim}} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \overset{R_3 \leftarrow R_3 - 3R_2}{\sim} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(c) Since there is a row entirely of zeros and there are no contradictions, there are an infinite number of solutions. Since we haven't solved for x_2, we parametrize in terms of x_2: $x_1 = 3 - 2x_2$. In other words: $\left\{ \begin{array}{l} x_1 = 3 - 2t \\ x_2 = t \\ x_3 = 0, \ t \in \mathbb{R} \end{array} \right\}$

$x_3 = 0$

$x_2 = x_2$

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② (a) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 0 \\ 1 & 2 & 3 & 3 \end{bmatrix} \underset{\substack{R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - R_1}}{\sim}$ (b) $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 3 & 0 \end{bmatrix} \underset{R_3 \leftarrow R_3 - 3R_2}{\sim} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 18 \end{bmatrix} = REF(A)$

REF

Since the ~~RREF~~ contains the contradiction $0 = 18$, there are no solutions.

(b) (continued) $\overset{R_3 \leftarrow R_3/18}{\sim} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \underset{R_1 \leftarrow R_1 - 3R_3}{\overset{R_2 \leftarrow R_2 + 6R_3}{\sim}} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = RREF(A)$

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③ $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 0 \\ 1 & 0 & 3 & 3 \end{bmatrix} \begin{array}{l} R_2 \leftarrow R_2 - 2R_1 \\ R_3 \leftarrow R_3 - R_1 \end{array} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -6 \\ 0 & -2 & 3 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -2 & 3 & 0 \\ 0 & 0 & 1 & -6 \end{bmatrix} \ R_2 \leftarrow -\frac{1}{2} R_2 \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 1 & -6 \end{bmatrix}$

$R_2 \leftarrow R_2 + \frac{3}{2} R_3 \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -6 \end{bmatrix} \ R_1 \leftarrow R_1 - 2R_2 \begin{bmatrix} 1 & 0 & 0 & 21 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -6 \end{bmatrix}$

Thus, the unique solution to the system is $x_1 = 21$, $x_2 = -9$, $x_3 = -6$
