Math. 362-02
Fall, 2016
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Fourth Exam

Thursday, November 17, 2016

This exam is closed book, but you may use calculators that do not have computer algebra systems. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Check your work carefully. Each part of each problem is worth 16 points, and 4 points are free. You may keep this exam sheet upon leaving.

1. Consider:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & -1 & 1 \\ 0 & 3 & 2 & 5 \\ 2 & 1 & 4 & 3 \end{bmatrix}.$$

   (a) Find a basis for the null space of A.

   (b) Find a basis for the column space of A.

   (c) Find a basis for the row space of A.

2. Consider the following two bases.

$$B = \left\{ \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \right\} \quad \text{and} \quad B' = \left\{ \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} \right\}.$$

   (a) Find the transition matrix $P_{B \to B'}$.

   (b) Compute the coordinate vector $[w]_{B'}$, where $[w]_B = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$.

   (c) Compute the representation of w in the standard basis.

   (Hint: There is a simple exact answer that is not too hard to obtain without a calculator. However, be very careful in your computations.)