## Fourth Exam

Tuesday, December 1, 2015
This exam is closed book, but you may use calculators. The exam should be done on your own paper. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. Each entire problem is worth 25 points. You may leave when you finish, and you may keep this sheet with the questions.

## 1. Consider

$$
A=\left[\begin{array}{lllll}
0 & 1 & 0 & 2 & 0  \tag{1}\\
0 & 0 & 1 & 3 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

(a) Write down the leading variables and free variables corresponding to $A$.
(b) Write down a basis for the null space of $A$.
2. Write down a basis for the row space of the matrix $A$ in (1).
3. Write down the eigenvalues and corresponding eigenvectors of the matrix $A$ in (1). Also, answer the following:
(a) Are there multiple eigenvalues?
(b) Does the set of all eigenvectors span $\mathbb{R}^{5}$ ?
4. Write down the eigenvalues and one of the eigenvectors of the following matrix:

$$
A=\left[\begin{array}{rrr}
2 & -1 & 0 \\
-1 & 2 & -1 \\
0 & -1 & 2
\end{array}\right]
$$

