Math. 362-02
Fall, 2015
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## First Exam

Thursday, September 24, 2015
This exam is closed book, but you may use calculators. Make sure your name is on all pages. Show all work, and show it in a logical and organized manner: You will be graded on what you show, in addition to your answer. The first problem is worth 50 points, the second problem is worth 10 points, and the third and fourth problems are each worth 20 points.

1. Consider the matrix $A=\left[\begin{array}{rrr}1 & 2 & 3 \\ -1 & -1 & -2 \\ 1 & 1 & 4\end{array}\right]$.
(a) Find $A^{-1}$ exactly using either Gaussian elimination or Gauss-Jordan elimination. Show all of the steps in your computation.
(b) Check that you have the correct $A^{(-1)}$ by computing $A^{-1} A$ (which should be the same as $A A^{-1}$ ). Show your computation.
(c) Use $A^{-1}$ to compute the solution to the linear system of equations

$$
\begin{array}{rr}
x_{1}+2 x_{2}+3 x_{3}=-1 \\
-x_{1}-x_{2}-2 x_{3}=2 \\
x_{1}+x_{2}+4 x_{3}=1
\end{array}
$$

Show your computation.
(d) Check your solution $\left(x_{1}, x_{2}, x_{3}\right)$ by matrix-vector multiplication. Show your computation.
2. Suppose we have a linear transformation $T: \mathbb{R}^{3} \rightarrow \mathbb{R}^{4}$, such that

$$
\begin{aligned}
& T\left(e_{1}\right)=(1,2,3,4) \\
& T\left(e_{2}\right)=(-4,-3,-2,-1), \text { and } \\
& T\left(e_{3}\right)=(-1,1,-1,1)
\end{aligned}
$$

Write down a matrix $A$ such that, for $x \in \mathbb{R}^{3}, T(x)=A x$.
3. Write down the set of solutions of the system of equations

$$
\begin{aligned}
& x_{1}+2 x_{2}+3 x_{3}=-1 \\
&-x_{1}-x_{2}-2 x_{3}= 2
\end{aligned}
$$

in parametric form. Show your work.
4. If $A=\left[\begin{array}{rrr}1 & 2 & 3 \\ -1 & -1 & -2\end{array}\right]$, compute $A A^{T}$ and $A^{T} A$.

