Math. 350-01, Summer, 2015, First exam answers, page 1

(1) (a) $\int \frac{dv}{v-1} = \int -9.8\,dt$.

$\ln|v-1| = -9.8t + C$

$\therefore v = 1 + ke^{-9.8t}$.

$v(0) = 49 \Rightarrow 49 = 1 + k \Rightarrow k = 48$ $\boxed{\therefore v(t) = 1 + 48e^{-9.8t}}$

(b) $v(.5) = 1 + 48e^{-9.8(0.5)} \approx$ $\boxed{1.36 \text{ meters per second}}$

(c) $\lim_{t\to\infty} v(t) =$ $\boxed{1 \text{ meter per second}}$

(2) (a) Nonlinear, because of the e^y term.

(b) linear.

(c) linear.

(d) Nonlinear, because of the y^2 term

(3) $\frac{dy}{dt} + \frac{1}{t}y = t$. $u' = \cancel{-}\frac{1}{t}u \Rightarrow \int \frac{du}{u} = \int \frac{dt}{t} \Rightarrow \ln|u| = \ln|t| + C$

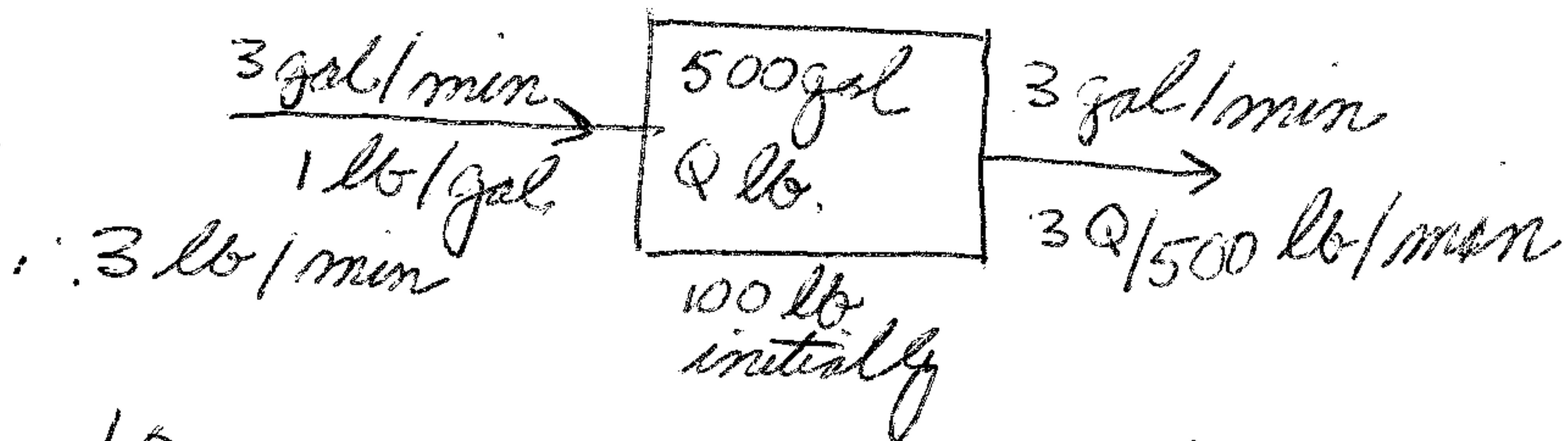
Thus, $u = t$ will do.

$(ty)' = t^2 \Rightarrow ty = \frac{t^3}{3} + C \Rightarrow$ $\boxed{y(t) = \frac{t^2}{3} + \frac{C}{t}}$

$y(\cancel{0}) = \frac{1}{3} + C = 1 \Rightarrow C = 2/3$ $\boxed{\therefore y(t) = \frac{t^2}{3} + \frac{2}{3}\left(\frac{1}{t}\right)}$

4

a



$$\frac{dQ}{dt} = 3 - \frac{3}{500}Q \text{ lb/min}; \quad \frac{dQ}{dt} = \frac{-3}{500}(Q-500) \text{ lb/min}$$

$$\int \frac{dQ}{Q-500} = \int \frac{-3}{500}dt \Rightarrow \ln|Q-500| = \frac{-3}{500}t + C$$

$$\Rightarrow Q(t) = 500 + ke^{\frac{-3}{500}t}. \quad Q(0) = 100 = 500 + k$$

$$\Rightarrow k = -400 \Rightarrow \boxed{Q(t) = 500 - 400e^{-\frac{3}{500}t}}$$

---

b $\displaystyle\lim_{t\to\infty} Q(t) = \boxed{500 \text{ lb}}$