① The characteristic equation is $r^2+6r+13=0$,

giving $r = -\frac{6}{2} \pm \frac{\sqrt{6^2-4(13)}}{2} = -3 \pm \frac{\sqrt{-16}}{2} = -3 \pm 2i$.

Thus, the solution to the homogeneous equation is:

$$y_h = e^{-3t}\left(C_1\cos(2t)+C_2\sin(2t)\right).$$

A particular solution is of the form $A\cos t + B\sin t$, so:

$$\begin{array}{lrl} & 13y: & 13A\cos t + 13B\sin t \\ + & 6y': & 6B\cos t - 6A\sin t \\ + & y'' & -A\cos t - B\sin t \\ \hline = & & (12A+6B)\cos(t) + (-6A+12B)\sin(t) = \sin(t). \end{array}$$

Thus $-6A+12B = \cancel{0}\ 1 \Rightarrow \cancel{A = 2B}$.

$12A+6B = 0 \Rightarrow B = -2A$, whence $-6A-24A = 1$

$-30A = 1 \Rightarrow A = \frac{-1}{30}$, $B = \frac{1}{15}$, so

$$\boxed{y(t) = e^{-3t}\left[C_1\cos(2t)+C_2\sin(2t)\right] - \frac{1}{30}\cos(t) + \frac{1}{15}\sin(t)}$$

---

② ⓐ $R = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$ $\tan(\delta) = \frac{1}{-1} = -1$, and,

since $\delta$ is a second quadrant angle, $\delta = \frac{3\pi}{4}$.

$$\boxed{\therefore\ y(t) = \sqrt{2}\cos\left(t - \frac{3\pi}{4}\right)}$$

ⓑ The amplitude is $\sqrt{2}$.

ⓒ The period is $2\pi$

ⓓ The phase shift is $\frac{3\pi}{4}$.

3 $y''+y=\sin(t)$, $y(0)=0$, $y'(0)=0$.

The characteristic equation is $r^2+1=0 \Rightarrow r=\pm i$

$\Rightarrow y_h(t)=C_1\cos(t)+C_2\sin(t)$.

Since $\sin(t)$ solves the homogeneous equation, we

assume $y_p(t)=At\cos(t)+Bt\sin(t)$

$y_p'(t)=A\cos(t)+B\sin(t)+t\left(-A\sin(t)+B\cos(t)\right)$

$y_p''(t)=-2B\cos(t)-2A\sin(t)+t\left(-A\cos(t)-B\sin(t)\right)$

Thus, $y_p''+y_p=-2B\cos(t)-2A\sin(t)=\sin(t)$.

Therefore, $-2A=1 \Rightarrow A=-\frac{1}{2}$, $B=0$.

Thus, $y(t)=C_1\cos(t)+C_2\sin(t)-\frac{1}{2}t\cos(t)$.

$y(0)=\boxed{C_1=0}$

$y'(t)=-C_1\sin(t)+C_2\cos(t)-\frac{1}{2}\cos(t)+\frac{1}{2}t\sin(t)$

$y'(0)=C_2-\frac{1}{2}=0 \Rightarrow C_2=\frac{1}{2}$.

Thus, $\boxed{y(t)=\frac{1}{2}\sin(t)-\frac{1}{2}t\cos(t)}$.

---

- The three terms in the D.E. are force in dynes.
- The spring constant is in dynes/centimeter.
- The mass is in grams.