1. The characteristic equation is: $r^2+4r+13=0$, so

$$r = \frac{-4}{2} \pm \frac{\sqrt{4^2-4(13)}}{2} = \frac{-4}{2} \pm \frac{\sqrt{-36}}{2} = -2 \pm 3i.$$

Thus, the general solution to the homogeneous equation is $y_h(t) = C_1 e^{-2t}\cos(3t) + C_2 e^{-2t}\sin(3t)$.

A particular solution will be of the form:

$y_p = A\cos t + B\sin t$. Thus

$$\begin{array}{ll} 13y_p & 13A\cos t + 13B\sin t \\ +4y_p' & 4B\cos t - 4A\sin t \\ +y_p'' & -A\cos t - B\sin t \\ \hline = & (12A+4B)\cos(t) + (-4A+12B)\sin(t) = \sin(t) \end{array}$$

Thus, $12A+4B = 0 \Rightarrow A = \frac{-1}{3}B$

$-4A+12B = 1 \Rightarrow \frac{4}{3}B + \frac{36}{3}B = \frac{40}{3}B = 1 \Rightarrow \boxed{B = \frac{3}{40}, A = \frac{-1}{40}}$.

$\Rightarrow$ Thus, $\boxed{y_p(t) = \frac{-1}{40}\cos(t) + \frac{3}{40}\sin(t)}$.

Thus, the general solution is

$$y(t) = C_1 e^{-2t}\cos(3t) + C_2 e^{-2t}\sin(3t) - \frac{1}{40}\cos(t) + \frac{3}{40}\sin(t)$$

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2. (a) $R = \sqrt{1^2+(-1)^2} = \sqrt{2}$. $\tan(\delta) = \frac{-1}{1} = -1$. Since δ is a ~~second~~ ~~third~~ fourth quadrant angle, $\delta = -\pi/4$, so

$\cos(t) - \sin(t) = \sqrt{2}\cos(t+\pi/4)$

(b) The amplitude is $\sqrt{2}$

(c) The period is 2π.

(d) The phase shift is $-\pi/4$.

3 $my'' + ky = f(t)$; $y'' + \frac{.1}{.1} y = \sin(t)$, $y(0) = 0$, $y'(0) = 0$.

The characteristic equation is $r^2 + 1 = 0 \Rightarrow r = \pm i$

$\Rightarrow y_h(t) = C_1 \cos(t) + C_2 \sin(t)$.

Since $\sin(t)$ solves the homogeneous equation, we

assume $y_p(t) = At\cos(t) + Bt\sin(t)$

$y_p'(t) = A\cos(t) + B\sin(t) + t(-A\sin(t) + B\cos(t))$

$y_p''(t) = -2B\cos(t) - 2A\sin(t) + t(-A\cos(t) - B\sin(t))$

Thus, $y_p'' + y_p = -2B\cos(t) - 2A\sin(t) = \sin(t)$.

Therefore, $-2A = 1 \Rightarrow A = -\frac{1}{2}$, $B = 0$.

Thus, $y(t) = C_1\cos(t) + C_2\sin(t) - \frac{1}{2}t\cos(t)$.

$y(0) = \boxed{C_1 = 0}$

$y'(t) = -C_1\sin(t) + C_2\cos(t) - \frac{1}{2}\cos(t) + \frac{1}{2}t\sin(t)$

$y'(0) = C_2 - \frac{1}{2} = 0 \Rightarrow C_2 = \frac{1}{2}$.

Thus, $\boxed{y(t) = \frac{1}{2}\sin(t) - \frac{1}{2}t\cos(t)}$.