



② Dividing both sides by t gives: $y' + \frac{1}{t}y = \frac{2}{t}$.

Thus, an integrating factor is: $\mu = e^{\int \frac{1}{t} dt} = e^{\ln(t)} = t$,

and $(ty)' = 2$. (note: this might be seen directly from the original equation.) Thus,

$$ty = 2t + C \Rightarrow y(t) = 2 + \frac{C}{t}$$

$$1 = y(1) = 2 + \frac{C}{1} \Rightarrow C = -1 \Rightarrow \boxed{y(t) = 2 - \frac{1}{t}}$$

③ Solving the IVP: $\frac{dv}{dt} = -\frac{1}{20}(v-196) \Rightarrow \int \frac{1}{v-196} dv dt = \int -\frac{1}{20} dt$

$$\Rightarrow \ln|v-196| = -\frac{1}{20}t + C \Rightarrow v = 196 + Ce^{-\frac{1}{20}t}$$

$$v(0) = 20 = 196 + C \Rightarrow C = -196 \Rightarrow \boxed{v(t) = 196(1 - e^{-\frac{1}{20}t})}$$

(a) Let T be the time when the object reaches 25% of the limiting velocity. $v(T) = .25(196) = 196(1 - e^{-\frac{T}{20}})$

$$\Rightarrow 1 - e^{-\frac{T}{20}} = .25 \Rightarrow e^{-\frac{T}{20}} = .75 = \frac{-T}{20} = \ln(.75)$$

$$\Rightarrow T = -20 \ln(.75) \approx \boxed{5.75 \text{ seconds}}$$

(b) The distance travelled is: $\int_0^T v(t) dt = 196 \int_0^T (1 - e^{-t/20}) dt$

$$= 196 \left[t + 20e^{-t/20} \right] \Big|_{t=0}^T \approx 196 \left[5.75 + 20e^{-5.75/20} - 20 \right]$$

$$\approx \boxed{147.54 \text{ meters}}$$