$$\mathcal{L}(y'') + \mathcal{L}(4y) = \mathcal{L}\left(u_\pi(t) - u_{2\pi}(t)\right)$$

$$s^2 Y(s) - 1 + 4Y(s) = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}.$$

$$(s^2+4)Y = 1 + \frac{1}{s}\left(e^{-\pi s} - e^{-2\pi s}\right).$$

$$Y = \frac{1}{s^2+4} + \left[\frac{1}{s(s^2+4)}\right]\left(e^{-\pi s} - e^{-2\pi s}\right)$$

$$\frac{1}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4},$$ whence

$$1 = A(s^2+4) + (Bs+C)s$$

$$1 = (A+B)s^2 + Cs + 4A \Rightarrow 4A = 1, \boxed{A = \tfrac{1}{4}}$$

$$A+B=0 \Rightarrow \boxed{B = -1/4}, \quad \boxed{C=0}$$

Thus,

$$Y = \frac{1}{s^2+4} + \left(\frac{1/4}{s} - \frac{1}{4}\frac{s}{s^2+4}\right)\left[e^{-\pi s} - e^{-2\pi s}\right]$$

Using formula 5 with $\frac{1}{s^2+4} = \frac{1}{2}\frac{2}{s^2+4}$,

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right) = \frac{1}{2}\sin(2t).$$ Also,

$$\mathcal{L}^{-1}(1/4/s) = 1/4, \text{ and } \mathcal{L}^{-1}\left(-\frac{1}{4}\frac{s}{s^2+4}\right) = -\frac{1}{4}\cos(2t)$$ (formula 6)

Combining and using formula 13, we obtain:

$$\boxed{y(t) = \frac{1}{2}\sin(2t) + \left[\frac{1}{4} - \frac{1}{4}\cos(2(t-\pi))\right]u_\pi(t) - \left[\frac{1}{4} - \frac{1}{4}\cos(2(t-2\pi))\right]u_{2\pi}(t)}$$