① $y(x)=\sum_{n=0}^{\infty}a_n x^n.$ $y'(x)=\sum_{n=1}^{\infty}n a_n x^{n-1}=\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n.$

$y''(x)=\sum_{n=2}^{\infty}n(n-1)a_n x^{n-2}$, so

$\cancel{xy}\ (x-1)y''(x)=\sum_{n=2}^{\infty}n(n-1)a_n x^{n-1}-\sum_{n=2}^{\infty}n(n-1)a_n x^{n-2}$

$$=\sum_{n=1}^{\infty}(n+1)n a_{n+1}x^n-\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n.$$

Thus, $(x-1)y''+y'+y$

$$=\sum_{n=1}^{\infty}(n+1)n a_{n+1}x^n-\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n+\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n+\sum_{n=0}^{\infty}a_n x^n$$

$$=\sum_{n=1}^{\infty}\left[(n+1)n a_{n+1}-(n+2)(n+1)a_{n+2}+(n+1)a_{n+1}+a_n\right]x^n$$

$$-2(1)a_2+a_1+a_0=0.$$

Thus, $\boxed{a_2=\frac{a_0+a_1}{2}}$, and,

$$\boxed{a_{n+2}=\frac{(n+1)^2a_{n+1}+a_n}{(n+2)(n+1)}\quad\text{for } n\ge 1}$$

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② $a_0=1$, $a_1=0$, so $a_2=\frac{1+0}{2}=\frac{1}{2}$,

$a_3=\frac{2^2(\frac{1}{2})+0}{6}=\frac{1}{3}$, $a_4=\frac{3^2(\frac{1}{3})+\frac{1}{2}}{12}=\frac{7}{24}$

Thus,

$$\boxed{y(x)\approx 1+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{7}{24}x^4}$$