Math. 350-01
Fall, 2013
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Third Exam Answers

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1. Using 10 meters/second/second as the acceleration of gravity, 10kg corresponds to $10(10) = 100$ Newtons of force. Thus, the spring constant is

$$k = 100/0.40 = 225\text{Newtons/meter}.$$

The equation of motion is thus

$$10y'' + 250y = 0, \quad \text{with} \;\; y(0) = .5, \;\; y'(0) = .25.$$

Dividing the differential equation by 10 gives

$$y'' + 25y = 0,$$

with characteristic equation

$$r^2 + 25 = 0, \qquad \text{with solutions } r = \pm 5i.$$

Thus, the general solution to the differential equation is

$$y(t) = C_1 \cos(5t) + C_2 \sin(5t).$$

We have

$$y(0) = C_1 = \tfrac{1}{2}, \quad \text{and } y'(t) = -5C_1 \sin(5t) + 5C_2 \cos(5t) \implies y'(0) = 5C_2 = \tfrac{1}{4} \implies C_2 = \tfrac{1}{20}.$$

2. $R = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{20}\right)^2} \approx 0.5025$ meters ≈ 50 centimeters, and, since both C_1 and C_2 are positive,

$$\delta = \arctan\left(\tfrac{1}{20}/\tfrac{1}{2}\right) = \arctan(0.1) \approx 0.0997 \text{ radians},$$

so

$$y(t) \approx 0.5025 \cos(5t - 0.0997) \quad \text{meters}.$$

Thus, the natural frequency is $\omega = 5$ radians per second, the period is $2\pi/5 \approx 1.26$ seconds, the amplitude (maximum deviation from the resting position) is about $R \approx 50$ centimeters, and the phase shift is $\delta \approx 0.0997$ radians.

Note: It is more accurate (and indeed is also correct) if 9.8 meters per second per second is used as the acceleration of gravity. However, the numbers are not as simple then.