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1. $y(x)=\sum_{n=0}^{\infty} a_n x^n,\ y'(x)=\sum_{n=1}^{\infty} n a_n x^{n-1}=\sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$

(a) (70pts) $y''=\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$

$xy=\sum_{n=0}^{\infty} a_n x^{n+1}=\sum_{n=1}^{\infty} a_{n-1}x^n.$

Thus, we have

$$y''+xy=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n+\sum_{n=1}^{\infty} a_{n-1}x^n$$

$$=2(1)a_2+\sum_{n=1}^{\infty}\left[(n+2)(n+1)a_{n+2}+a_{n-1}\right]x^n.$$

We obtain:

$2a_2=0 \Rightarrow a_2=0$, and the general recursion:

$$a_{n+2}=\frac{-a_{n-1}}{(n+1)(n+2)},\quad n\geq 1.$$

$\underline{n=1}$: $a_3=\frac{-a_0}{(2)(3)}=\frac{-a_0}{6}$

$\underline{n=2}$: $a_4=\frac{-a_1}{4(3)}=\frac{-a_1}{12}$

$n=3$: $a_5=\frac{-a_2}{(5)(4)}=0$. (Thus, the degree 5 term is 0.

The general solution is thus:

$$\boxed{y(x)=a_0\left(1-\frac{x^3}{6}+\cdots\right)+a_1\left(x-\frac{x^4}{12}+\cdots\right)}.$$

(b) $y(0)=a_0=1$, and $y'(0)=a_1=0$.

Therefore, the terms up to and including degree 5 of the solution to the initial value problem are:

$$\boxed{y(x)\approx 1-\frac{x^3}{6}}$$

2. This is an Euler equation, so assume a solution

(30pts) of the form $y(x) = x^r$.

Plugging in, we get $x^2(r)(r-1)x^{r-2} - 2x^r$

$= x^r[r(r-1) - 2] = 0.$

Unless $x = 0$, we have $r^2 - r - 2 = 0$,

that is, $(r-2)(r+1) = 0 \Rightarrow r = 2$ or $r = -1$.

The general solution to the equation is thus:

$$\boxed{y(x) = c_1 x^2 + \frac{c_2}{x}}.$$