Math. 350-03, Friday 2007/11/02 - Third exam answers (1)

(1) (a) $y_1 = 1 + (.25)1^2 = 1.25$

$y_2 = 1.25 + (.25)(1.25)^2 \approx 1.640625$

$y_3 = 1.64 + (.25)(1.64)^2 = 2.3124$

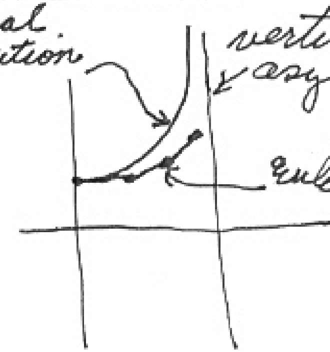
$y_4 = 2.31 + (.25)(2.31)^2 = \boxed{3.644025}$

(b) $\int \frac{dy}{y^2} = \int dt$, so $-\frac{1}{y} = t + C$ so $y = \frac{1}{K-t}$ for K an arbitrary constant

$y(0) = 1 \Rightarrow 1 = \frac{1}{K} \Rightarrow K = 1.$

$\Rightarrow \boxed{y(t) = \frac{1}{1-t}}$ $y(1)$ is undefined $y(.75) = \frac{1}{\frac{1}{4}} = 4$

(c) The exact solution at $t = .75$ is larger than the approximate solution. The reason is because the derivative is an increasing function of y, so the function is concave up, so tangent line approximations to y lie below y.



Math. 350-03, Friday 2007/11/02 – Third exam answers (2)

(2) (9) The characteristic equation is:

$$\alpha^2+5\alpha+4=0, \quad (\alpha+4)(\alpha+1)=0,$$ so the general solution is:

$$y(t)=C_1e^{-4t}+C_2e^{-t}.$$

$$y(0)=1=C_1+C_2$$

$$y'(t)=-4C_1e^{-4t}-C_2e^{-t}$$

$$y'(0)=-4C_1-C_2=0 \Rightarrow C_2=-4C_1,$$ so

$$1=C_1-4C_1=-3C_1 \Rightarrow C_1=-1/3,\ C_2=4/3.$$

Thus $\boxed{y(t)=-\frac{1}{3}e^{-4t}+\frac{4}{3}e^{-t}}$

---

(10) We already know the solution to the homogeneous equation from part (9). The particular solution can be of the form $y_p(t)=A\cos t+B\sin t$.

$$y_p'=-A\sin t+B\cos t$$

$$y_p''=-A\cos t-B\sin t$$

So:

$$y_p''+5y_p'+4y_p=(-A+5B+4A)\cos t+(-B-5A+4B)\sin t,$$

which gives:

$$\begin{cases}3A+5B=0\\-5A+3B=1\end{cases} \Rightarrow B=\frac{-3}{5}A \Rightarrow \cancel{-5B+}$$

$$-5A+3\left(\frac{-3}{5}A\right)=1$$

$$\Rightarrow \frac{-25-9}{5}A=1 \Rightarrow A=-\frac{5}{34};\ B=\frac{3}{34},$$

so the general solution is:

$$\boxed{y(t)=C_1e^{-4t}+C_2e^{-t}-\frac{5}{34}\cos(t)+\frac{3}{34}\sin(t)}$$