① The D.E. is separable:

$$\int (3+2y)\,dy = \int (2-e^x)\,dx$$

$3y+y^2 = 2x - e^x + C$. $y(0)=0 \Rightarrow 3\cdot 0 + 0^2 = 2\cdot 0 - e^0 + C$

i.e. $0 = -1 + C \Rightarrow C = 1 \Rightarrow 3y + y^2 = 2x - e^x + 1$.

In standard form, we have:

$y^2 + 3y - (2x - e^x + 1) = 0$. Using the quadratic formula gives:

$$y = \frac{-3 \pm \sqrt{9 + 4(2x - e^x + 1)}}{2}$$, where the

positive branch is taken because of the initial condition. If $u = 2x - e^x + 1$, then y is an increasing function of u, so y attains its maximum where u does. However $\frac{du}{dx} = 2 - e^x > 0$ for $x < \ln(2)$,

and $\frac{du}{dx} < 0$ for $x > \ln(2)$. Thus, y attains a maximum at $x = \ln(2)$.

(1) We will write a differential equation for the total quantity $Q(t)$ in grams in the lake as a function of time t years. We have:

$$\frac{dQ}{dt} = 10{,}000\left\{100\left(1-t/10\right)\right\} - \frac{10{,}000\,Q}{10{,}000}$$

$\frac{dQ}{dt} + Q = 10^6\left(1-t/10\right)$. An integrating factor is: $u(t) = e^t$.

$$\left(e^t Q\right)' = \left(10^6 - 10^5 t\right)e^t, \text{ so}$$

$$e^t Q = 10^6 \int e^t\,dt - 10^5 \int t e^t\,dt = 10^6 e^t - 10^5 t e^t + 10^5 e^t + C$$

so $Q(t) = 10^6 - 10^5 t + 10^5 + Ce^{-t}$, so

$c(t) = \frac{Q(t)}{10^4} = 10^2 - 10t + 10 + \tilde{C}e^{-t}$ where $c(t)$ is the concentration at time t.

$c(0) = 100 = 100 + 10 + \tilde{C} \Rightarrow \tilde{C} = -10$

so $c(t) = 100 - 10t + 10\left(1-e^{-t}\right)$.

Thus, the concentration after 10 years is:

$c(10) = 10\left(1-e^{-10}\right) \approx 10$ grams per cubic meter.