Math. 350-01
Fall, 2002
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First Exam
Tuesday, October 15

This exam is closed book, but with the computer on. Make sure your name is on all pages, and put your name in a text cell on any Mathematica notebooks before you print them.

1. (20 points) State whether each of the following differential equations is linear or nonlinear. Explain why in each case. Also state the order of each equation.

   (a) $e^t \frac{d^2y}{dt^2} + y = \sin(t)$
   (b) $y \frac{dy}{dt} + t = 1$

2. (30 points) Use pencil and paper to derive the solution to the initial value problem

$$\frac{dy}{dt} + \frac{1}{t} y = t^2, \quad y(1) = 1.$$

3. (50 points) At a given level of effort, it is reasonable to assume that the rate at which fish are caught depends on the population y: The more fish there are, the easier it is to catch them. Thus we assume that the rate at which fish are caught is given by Ey, where E is a positive constant with units of 1/time, that measures the total effort made to harvest the given species of fish. To include the effect of harvesting, the logistic equation is replaced by

$$\frac{dy}{dt} + r\left(1 - \frac{y}{K}\right) y - Ey.$$

   For each of the following r, K, and E, find the equilibrium points, and say whether the equilibria are stable, asymptotically stable, or unstable. Also, for each of the following r, K, and E, plot a direction field and explain it in terms of the equilibrium points. Label each equilibrium point on the direction field. (Relate your stability analysis to what you see in the direction fields.) Finally, interpret the results for each of the three sets of parameters in terms of the fisheries.

   (a) $K = 10,000$, $r = 0.2$, $E = 0.02$.
   (b) $K = 10,000$, $r = 0.2$, $E = 0.19$.
   (c) $K = 10,000$, $r = 0.2$, $E = 0.4$.

   (We will discuss this model further in class at a later date.)