(1) $\frac{\partial g}{\partial u} = 10(\sin(t)+e^u)^9 e^u$; $\frac{\partial g}{\partial t} = 10(\sin(t)+e^u)^9\cos(t)$

(2) $f_x = 2xy \quad f_y = x^2$; $f_x(1,-1) = -2$; $f_y(1,-1) = 1$. $f(1,-1) = -1$

An equation for the tangent plane is thus:

$$z = f(1,-1) + f_x(1,-1)(x-1) + f_y(1,-1)(y+1)$$

$$\boxed{z = -1 + (-2)(x-1) + (y+1)} = -2x + y + 2 = z$$

(3) $\nabla f(1,-1) = (-2,1)$. $D_{\vec{u}} f(1,-1) = \nabla f(1,-1) \circ \vec{u}$

$$= (-2,+1)\circ\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = \boxed{\frac{-2}{\sqrt{5}} + \frac{2}{\sqrt{5}} = 0}$$

(5) $\nabla f = (2x+y, x+2y)$. This gives $2x+y=0$, $x+2y=0$, from which $x=0, y=0$ is the only critical point.

$f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 1$, so

$D = f_{xx}f_{yy} - (f_{xy})^2 = 4-1 = 3 > 0$. Hence, $(0,0)$ corresponds to a local minimum.

(4) $f_x = \frac{\partial f}{\partial x} = 10(e^x+y)^9 e^x$. $\frac{\partial f}{\partial y} = 10(e^x+y)^9$

$\frac{dx}{dt} = -\sin(t)$, $\frac{dy}{dt} = \cos(t)$, so

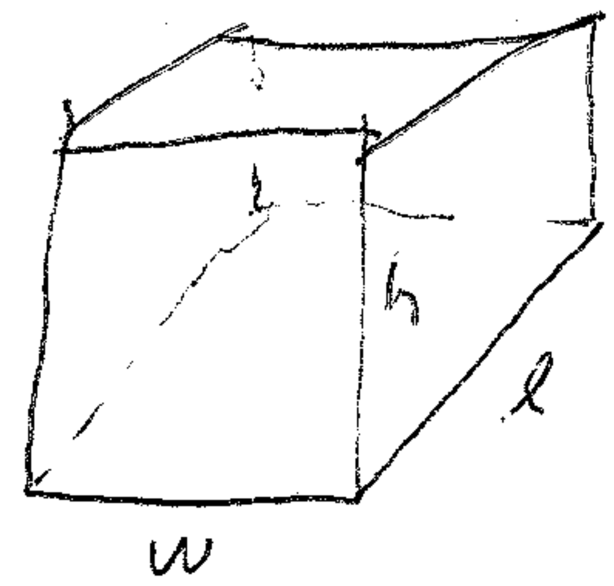
$$\frac{df}{dt} = 10e^x(e^x+y)^9 e^x(-\sin(t)) + 10(e^x+y)^9\cos(t)$$

$x(\frac{\pi}{2}) = 0$; $y(\frac{\pi}{2}) = 1$. This gives

$$\left.\frac{df}{dt}\right|_{t=\pi/2} = 10(1+1)^9(-1) + 10(1+1)^9(0) - $$

$$= -10(2^9) = \boxed{-5120}$$

(6)



Let w be the width of the front, l the length of a side, and h the height of the compartment. Then $lwh = 10$.

The total cost is
$$C = (200lh + 100wh + 50wl)\cdot 2$$
$$= 400lh + 200wh + 100wl.$$

Solving for l: $l = \frac{10}{wh}$, we get
$$C(w,h) = \frac{4000}{w} + 200wh + \frac{1000}{h}.$$
$$\frac{\partial C}{\partial w} = \frac{-4000}{w^2} + 200h = 0, \quad \frac{\partial C}{\partial h} = 200w - \frac{1000}{h^2} = 0.$$
Solving the first equation for h gives $h = \frac{20}{w^2}$, and plugging into the second equation gives
$$200w - \frac{1000}{\frac{400}{w^4}} = 200w - 2.5w^4 = 0,$$
whence $w(800 - 10w^3) = 0$, i.e. $w(80 - w^3) = 0$. Since $w \neq 0$, this gives $\boxed{w = \sqrt[3]{80} \approx 4.31 \text{ meters}}$, $\boxed{h = \frac{20}{w^2} \approx 1.08 \text{ meters}}$,
$$l = \frac{10}{wh} = \frac{10}{(\sqrt[3]{80})\left(\frac{20}{80^{2/3}}\right)} = \frac{1}{2}\sqrt[3]{80} \approx \boxed{2.15 \text{ meters}}$$

(7) $f = x + y$, $g = x^2 - y^2 = 1$

$\nabla f = (1,1)$, $\nabla g = (2x, -2y)$

$\nabla f = \lambda \nabla g \Rightarrow 1 = 2x\lambda,\ 1 = -2y\lambda \Rightarrow 2x\lambda = -2y\lambda$,

i.e. $2\lambda(x+y) = 0$, so either $\lambda = 0$, not possible since $2x\lambda = 1$, or $x + y = 0$. Thus, in all cases, $y = -x$.

Plugging into $x^2 - y^2 = 1$ gives $x^2 - x^2 = 1 = 0$, a contradiction.

Therefore, the problem has no maximum or minimum.