(1) Let W be the total weight in kilograms. Then

$$W = \iiint_{\mathcal{V}} 1.4\times10^{9} e^{-0.105(\rho-6400)}\, dV, \text{ where}$$

$\mathcal{V}$ is the volume between the sphere of radius 6400 and radius 6700. Using spherical coordinates, we get:

$$W = \int_{\rho=6400}^{6700}\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi} 1.4\times10^{9} e^{-0.105(\rho-6400)} \rho^{2}\sin\varphi\, d\rho\, d\theta\, d\varphi$$

$$= 1.4\times10^{9}\left\{\int_{\rho=6400}^{6700} \rho^{2} e^{-0.105(\rho-6400)}\, d\rho\right\}\left\{\int_{\theta=0}^{2\pi} d\theta\right\}\left\{\int_{\varphi=0}^{\pi}\sin\varphi\, d\varphi\right\}$$

$$= (1.4\times10^{9})(2\pi)(2)\int_{\rho=6400}^{6700} \rho^{2} e^{-0.105(\rho-6400)}\, d\rho$$

$$= (1.4\times10^{9})(4\pi)\left[\frac{-1}{.105}\rho^{2}e^{-.105(\rho-6400)} - \frac{2}{(.105)^{2}}\rho e^{-.105(\rho-6400)} - \frac{2}{(.105)^{3}} e^{-.105(\rho-6400)}\right]\Bigg|_{\rho=6400}^{6700}$$

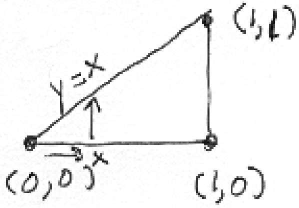
$$= (4\pi)(1.4\times10^{9})\left[-e^{-.105(300)}\left(\frac{6700^{2}}{.105} + \frac{2(6700)}{(.105)^{2}} + \frac{2}{(.105)^{3}}\right) + \left(\frac{6400^{2}}{.105} + \frac{2(6400)}{(.105)^{2}} + \frac{2}{(.105)^{3}}\right)\right]$$

$$\approx 4\pi(1.4\times10^{9})\left[-8.95\times10^{-6} + 3.91\times10^{9}\right]$$

$$\approx \boxed{6.88\times10^{18} \text{ kilograms}}$$

(2) (a)

$$I = \int_{x=0}^{1}\int_{y=0}^{x} (x^2+y^2)\,dy\,dx$$

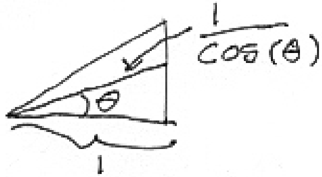


$$= \int_{x=0}^{1} \left[x^2 y + \frac{y^3}{3}\right]\Bigg|_{y=0}^{x} dx$$

$$= \int_{x=0}^{1} x^3 + \frac{x^3}{3}\,dx = \int_{x=0}^{1} \frac{4}{3}x^3\,dx = \frac{4}{3}\left[\frac{1}{4}x^4\right]\Bigg|_{x=0}^{1} = \boxed{\frac{1}{3}}$$

(b) Using polar coordinates, we have

$0 \le \theta \le \pi/4$ and $0 \le r \le \frac{1}{\cos(\theta)}$,



so

$$I = \int_{\theta=0}^{\pi/4}\int_{r=0}^{\sec(\theta)} r(r^2)\,dr\,d\theta$$

$$= \int_{\theta=0}^{\pi/4} \frac{r^4}{4}\Bigg|_{r=0}^{\sec(\theta)} d\theta = \frac{1}{4}\int_{\theta=0}^{\pi/4} \sec^4(\theta)\,d\theta$$

$$= \frac{1}{4}\left[\frac{2}{3}\tan\theta + \frac{1}{3}\sec^2\theta\tan\theta\right]\Bigg|_{\theta=0}^{\pi/4}$$

$$= \frac{1}{4}\left[\frac{2}{3}(1) + \frac{1}{3}(2)(1)\right] = \boxed{\frac{1}{3}}$$